The Unbearable Beingness of Light

– Dressing and Undressing Photons in Black Hole Spacetimes –

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ABSTRACT: Gravitational tidal forces acting on the virtual e^+e^- cloud surrounding a photon endow spacetime with a non-trivial refractive index. This has remarkable properties unique to gravitational theories including superluminal low-frequency propagation, in apparent violation of causality, and amplification of the renormalized photon field, in apparent violation of unitarity. Using the geometry of null congruences and the Penrose limit, we illustrate these phenomena and their resolution by tracing the history of a photon as it falls into the near-singularity region of a black hole.

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1. In general relativity, photons propagate along null geodesics, tracing out the causal structure of the background spacetime. Along these null paths, the proper time vanishes—in their frame, photons exist only in an instant. They have no history; no past, no future, no time in which to evolve or decay. They are caught forever in a single instant of time—the unbearable "beingness" of light [1].

Quantum field theory, however, profoundly changes this picture. Vacuum polarization envelops the photon in a cloud of virtual e^+e^- pairs. This dressed photon acquires an effective size, given by the Compton wavelength $\lambda_c = h/mc$ of the electron, and propagates as if it were an extended object, subject to gravitational tidal forces. This changes the nature of photon propagation as the phase velocity responds to the gravitational field and becomes frequency-dependent. In QED, photons see curved spacetime as an optical medium with a non-trivial refractive index.

The propagation of light in curved spacetime therefore exhibits all the usual optical phenomena associated with a refractive index $n(\omega)$, including dispersion and dissipation. The size of these effects is $\mathcal{O}(\alpha(\lambda_c/L)^2)$, where L is a typical curvature scale and α is the fine structure constant. However, there are surprises – as first discovered by Drummond and Hathrell [2], the low-frequency limit of the phase velocity can be superluminal, in apparent violation of causality. Moreover, the imaginary part of the refractive index can be negative [3], indicating an amplification rather than dissipation of light as it passes through curved spacetime, in apparent violation of unitarity.

Reconciling these phenomena with the fundamental principles of quantum field theory and general relativity has been the focus of our investigations of this subject [3–8]. We find that indeed causality and unitarity are respected, but at the expense of radical changes to many well-established principles of QFT in flat spacetime, notably the optical theorem and the range of theorems associated with analyticity, including the vital Kramers-Kronig dispersion relation. In particular, we find that the refractive index, now position-dependent, is governed by the whole past trajectory of the photon. In QFT, photons are liberated; they have a history, and a rich and fascinating one.

In this essay, we illustrate all these phenomena by following the history of a photon as it falls into a black hole, showing how the superluminal phase velocity is reconciled with causality and describing the effects of the gravitational tidal forces on the virtual e^+e^- cloud. Remarkably, we find that the photon can become undressed—the tidal forces squeeze the vacuum polarization cloud, reducing the screening and amplifying the renormalized quantum field amplitude as the photon approaches the singularity. This, then, is the photon's tale.

2. The most illuminating way to describe these effects in QED is in terms of an initial value problem for the photon field $A_\mu(x)$, which satisfies a wave equation incorporating the one-loop vacuum polarization given by the Feynman diagram in Fig. 1. In the eikonal approximation, the solution is

$$A_\mu^{(i)}(x) = g(u)^{-1/4}\mathcal{A}(u_0)e^{i\omega V}e^{i\omega\int_{u_0}^{u}du'(n(u';\omega)-1)_{ij}}\varepsilon_\mu^{(j)}(u) \ ,$$

where u is a null coordinate along the classical light ray γ, and V is the corresponding null coordinate labelling geodesics in the null congruence around γ. $\mathcal{A}(u_0)$ is the amplitude on the initial value surface $u = u_0$, $\varepsilon_\mu^{(i)}(u)$ is the polarization vector and $n_{ij}(u;\omega)$ is a position, polarization and frequency-dependent refractive index.

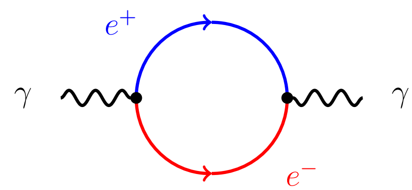

Figure 1. The one-loop Feynman diagram contributing to the vacuum polarization in QED.

A key observation is that to leading order in $(\lambda_c/L)^2$, we can replace the background spacetime by its Penrose limit [9] associated with the null geodesic γ. We have shown, using both worldline and conventional QFT methods [3–6], how the refractive index is governed by the geometry of geodesic deviation around γ. But this is precisely the property of the spacetime which is encoded in the Penrose limit. This can be seen by writing the metric around γ in null Fermi coordinates and making the Penrose rescaling $u \to u, v \to \lambda^2 v, x^i \to \lambda x^i$; truncating at $\mathcal{O}(\lambda^2)$ then recovers the Brinkmann plane wave metric $ds^2 = 2dudv - h_{ij}(u)x^i x^j du^2 + dx^i dx^i$, with the profile function identified as $h_{ij} = R_{uiuj}$ [10]. Since the geodesic deviation equation is written in terms of the Jacobi fields x^i as $d^2x^i/du^2 = R^i{}_{uju}x^j$, this isolates exactly the required curvature components. In QED, the expansion in λ is mirrored as an expansion of the Green functions in λ_c/L, which ensures that the gravitational curvature is relatively weak on the quantum scale.

The refractive index is expressed entirely in terms of geometric quantities related to geodesic deviation. Solving the Jacobi equation with "geodesic spray" boundary conditions at $u = u'$, as in Fig. 2, determines the Van Vleck-Morette matrix $\Delta_{ij}(u,u') = (u'-u)A^{-1}_{ji}(u,u')$. We have then shown that the refractive index of curved spacetime

$$\frac{d^2x^i}{du^2} = R^i{}_{uju}x^j \ ,$$

$$x^i(u) = A_{ij}(u,u')\frac{dx^j(u')}{du'} \ ,$$

$$A_{ij}(u',u') = 0, \quad \left.\frac{dA_{ij}(u,u')}{du}\right|_{u=u'} = \delta_{ij} \ .$$

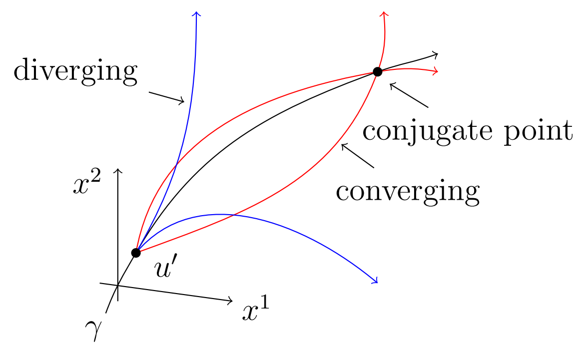

Figure 2. Null geodesic spray from a point u' on γ. On the left is the equation for geodesic deviation of the Jacobi fields $x^i(u)$ with a solution written in terms of the 2×2 matrix $A_{ij}(u,u')$. The boundary conditions on the spray are written in the last line. In the illustration on the right, geodesics in the spray at $u = u'$ diverge in one direction (blue) and converge in the other (red) to cross at a conjugate point.

in QED with scalar electrons, with initial value surface $u_0 \to -\infty$, is

$$n_{ij}(u;\omega) = \delta_{ij} - \frac{i\alpha}{2\pi\omega}\int_0^1 d\xi\,\xi(1-\xi)\int_{-\infty}^{u}\frac{du'}{(u-u')^2}e^{-\frac{im^2(u-u')}{2\omega\xi(1-\xi)}}\left[\Delta_{ij}(u,u')\sqrt{\Delta(u,u')} - \delta_{ij}\right] .$$

(A similar, more complicated, formula holds for spinor electrons.) Notice the key point that $n_{ij}(u,\omega)$ is given by an integral over the whole of the past worldline $u' < u$ of the photon. In curved spacetime, photons have a history, their local propagation characteristics being determined by the curvature throughout their path.

The refractive index has novel analyticity properties, related to the existence of conjugate points of the null geodesic congruence. This has far-reaching consequences [3] since it shows that amplitudes in curved spacetime have geometric as well as kinematic branch points and cuts, radically modifying many apparently fundamental theorems in quantum field theory and S-matrix theory. In particular, the Kramers-Kronig dispersion relation in curved spacetime becomes simply

$$n(u;\infty) = n(u;0) - \frac{1}{i\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\,n(u,\omega) \ .$$

In flat spacetime, hermitian analyticity would then imply that the principal value integral can be written in terms of $\mathrm{Im}\,n(u;\omega)$, which would be guaranteed by the optical theorem to be positive, forcing $n(\infty) < n(0)$. However, causality depends on the wavefront velocity (which is identified with the high frequency limit of the phase velocity [11]) not exceeding c, that is $n(\infty) \geq 1$. In flat spacetime, this would clearly be incompatible with a superluminal low-frequency value $n(0) < 1$. In curved spacetime, however, hermitian analyticity is violated due to the geometric cuts in the complex ω-plane in $n(u;\omega)$ and the conventional Kramers-Kronig relation fails.

Moreover, we also find instances where $\mathrm{Im}\, n(u;\omega)$ is negative, requiring a reappraisal of the optical theorem itself [7, 8]. The key point is that in curved spacetime, the optical theorem, which relates the imaginary part of scattering amplitudes to cross sections, only holds globally. For example, here we can express the total probability up to time u for the decay $\gamma \to e^+e^-$ as

$$P_{\gamma\to e^+e^-}(u) = 4\omega \int_{u_0}^{u} du'\, \mathrm{Im}\, n(u';\omega) \ .$$

which is manifestly positive. However, without u-translation invariance, the local version is no longer necessarily positive and does not represent a true decay rate, removing the apparent conflict between $\mathrm{Im}\, n(u;\omega) < 0$ and unitarity. The field may be locally amplified, but integrated over its whole past trajectory there must be a net attenuation.

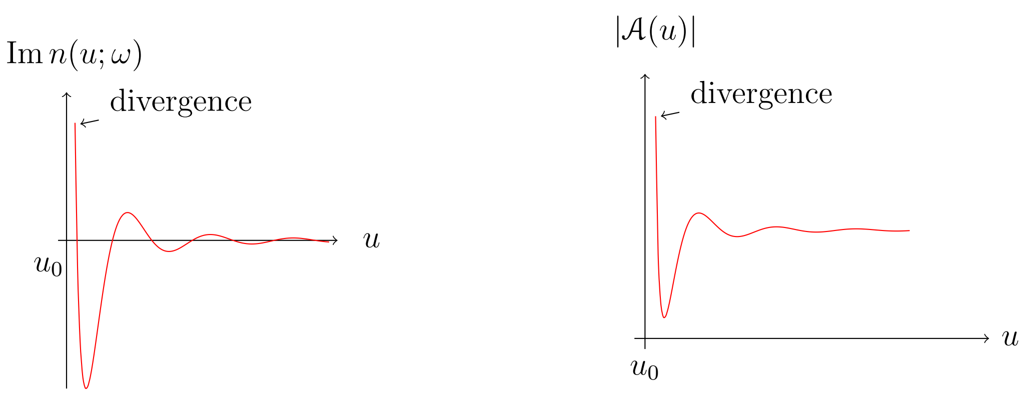

Figure 3. $\mathrm{Im}\, n(u;\omega)$ and the amplitude $|\mathcal{A}(u)|$ as a function of u in the near-flat spacetime region far from the black hole. This shows the characteristic oscillatory transient behaviour as the photon becomes dressed with its virtual e^+e^- cloud, screening the bare field and reducing the amplitude. In QED the initial value of the field is tuned to be divergent to give a finite $\mathcal{A}(u)$ away from the short-distance region close to u_0.

3. We are now ready to follow the evolution of the photon field as it falls towards the singularity of a black hole. Starting from an initial value surface u_0 far from the hole where spacetime is essentially flat, the imaginary part of the refractive index $\mathrm{Im}\, n(u;\omega)$ and the field amplitude $\mathcal{A}(u)$, for either polarization, have the form shown in Fig. 3.

Now consider a planar null geodesic in Schwarzschild spacetime with impact parameter b. In ref. [6], we developed new techniques for determining the Penrose limit for null geodesics in a general class of black hole spacetimes. We showed that the eigenvalues of the plane wave profile function h_{ij} are

$$h_{\pm} = \frac{1}{2} R_{\mu\nu} \hat{k}^{\mu} \hat{k}^{\nu} \pm \frac{3}{2} |\Psi_2|^{5/3} |K_s|^2 \ ,$$

where $\hat{k}^{\mu}$ is the tangent vector to γ, $\Psi_2 = -C_{\mu\nu\lambda\rho} \ell^{\mu} m^{\nu} \bar{m}^{\rho} n^{\rho} = -M/r^3$ is the Weyl tensor in the Newman-Penrose basis and K_s, with $|K_s|^2 = 2M^{-2/3} b^2$, is the Walker-Penrose conserved quantity [12, 13] associated with the null orbits. The Penrose limit therefore has $h_{11} = -h_{22} = 3Mb^2/r^5$.

An important simplification arises in the near-singularity limit, with $h_{11} = -h_{22} = 6/25u^2$, independent of M and b. This is a singular homogeneous plane wave. Other Penrose limits are readily calculated, e.g. the near-singularity limit of equatorial null orbits in the Kerr black hole are also homogeneous plane waves with identical h_{ij}, which is determined purely by the power-law nature of the singularity [14–16].

The refractive index in the near-singularity region now follows from the geodesic spray matrix found from the Jacobi equation with this h_{ij}, illustrated in Fig. 4.

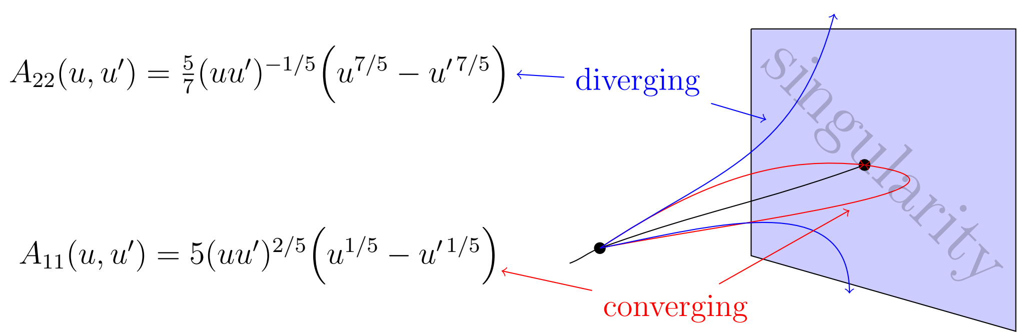

Figure 4. The behaviour of the null geodesic spray as the singularity is approached. In one plane the geodesics diverge (blue) while in the orthogonal plane geodesics converge with the singularity being a conjugate point (red).

The frequency dependence of $n(u;\omega)$ for fixed u is shown in Fig. 5. This shows explicitly how $\mathrm{Re}\, n(u;\omega) \to 1$ as $\omega \to \infty$ for both polarizations, including the polarization $\varepsilon^{(1)}$ which is superluminal at low frequency, ensuring causality is maintained. The converging polarization $\varepsilon^{(1)}$ also has $\mathrm{Im}\, n(u;\omega)$ negative, which would violate the conventional optical theorem but is consistent with our new curved spacetime formulation.

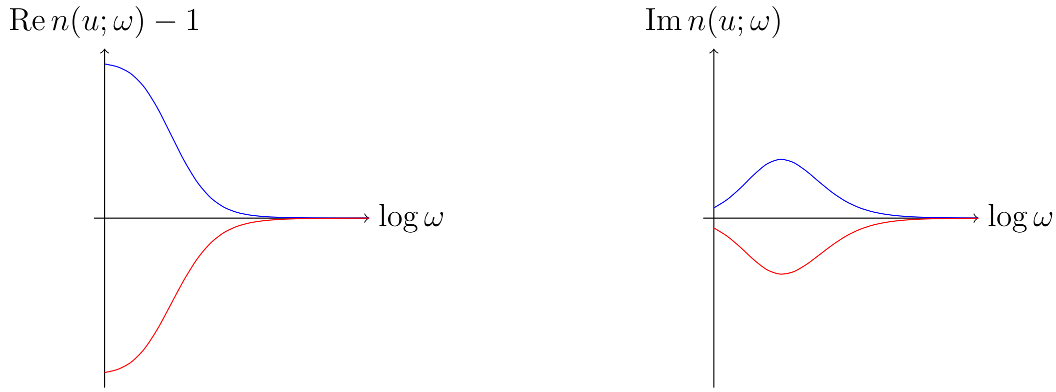

Figure 5. $\mathrm{Re}\, n(u;\omega)$ and $\mathrm{Im}\, n(u;\omega)$ as a function of frequency ω for fixed position u in the near singularity region of a Schwarzschild black hole. The red curves denote the polarization perpendicular to the orbital plane, corresponding to a converging null congruence, while the blue curves denote the polarization lying in the orbital plane, for which the null congruence is diverging. Note that $n(u;\omega) \to 1$ as $\omega \to \infty$, as required to maintain causality.

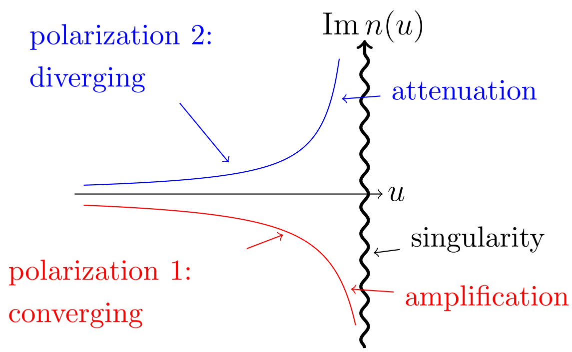

Figure 6. $\mathrm{Im}\, n(u;\omega)$ for the two polarizations in QED in a Schwarzschild black hole as u approaches the singularity at $u = 0$ from $u < 0$.

We now follow the photon as it approaches the singularity. The amplitude is given by

$$\frac{|\mathcal{A}^{(i)}(u)|}{|\mathcal{A}^{(i)}(u_1)|} = \exp\left[-\omega \int_{u_1}^{u} du' \;\; \mathrm{Im}\, n_{ii}(u;\omega)\right] \;,$$

where we compare with its value at some fixed point u_1 in the near-singularity region. The results are shown for the two polarizations in Fig. 6. As $|u|$ approaches zero, we find

$$\mathrm{Im}\, n_i(u;\omega) = \frac{\alpha}{12\pi\omega|u|} c_i \;,$$

with $c_1 = -0.155$ and $c_2 = 0.105$, which implies that the amplitude itself behaves as a power law:

$$|\mathcal{A}^{(i)}(u)| \sim |u|^{\alpha c_i/12\pi} \ .$$

The fate of the photon is therefore sealed as it falls into the singularity. For polarization $\varepsilon^{(2)}$, the curvature pulls virtual e^+e^- pairs from the bare field, intensifying the virtual cloud and screening the bare field so that its amplitude vanishes. For the other polarization, $\varepsilon^{(1)}$, the gravitational tidal forces squeeze the virtual cloud out of existence, undressing the photon and revealing the bare field itself. Its history has come full circle, from the initial transient dressing and the playing out of renormalization in real time as it propagates through curved spacetime, to its final destiny as the vacuum polarization cloud once more vanishes as it enters the black hole singularity.

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