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A proof of the strong cosmic censorship conjecture

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Abstract

The Penrose strong cosmic censorship conjecture asserts that Cauchy horizons inside dynamically formed black holes are unstable to remnant matter fields that fall into the black holes. The physical importance of this conjecture stems from the fact that it provides a necessary condition for general relativity to be a truly deterministic theory of gravity. Determining the fate of the Penrose conjecture in non-asymptotically flat black-hole spacetimes has been the focus of intense research efforts in recent years. In the present essay we provide a remarkably compact proof, which is based on Bekenstein's generalized second law of thermodynamics, for the validity of the intriguing Penrose conjecture in physically realistic (dynamically formed) curved black-hole spacetimes.

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The physically influential and mathematically elegant singularity theorems of Hawking and Penrose [1–3] have revealed the intriguing fact that the interior regions of dynamically formed black holes contain spacetime singularities. These are physically pathological regions in which the Einstein field equations lose their predictive power.

In order to guarantee the deterministic nature of general relativity as a successful theory of gravity, Penrose has put forward the cosmic censorship conjecture, according to which a mysterious “cosmic censor” prevents distant observers from being influenced by the disturbing singularities of highly curved spacetimes [2, 3]. This intriguing conjecture asserts, in particular, that Cauchy horizons inside physically realistic (dynamically formed) black holes, which mark the boundaries beyond which the Einstein field equations lose their predictive power, are singular. If true, this strong version of the Penrose conjecture would imply that physical observers are always restricted to live in spacetime regions in which general relativity is a physically successful and mathematically deterministic theory of gravity [2, 3].

In order to challenge the validity of the Penrose strong cosmic censorship conjecture, one may try to identify a physically realistic curved black-hole spacetime whose inner Cauchy horizon is stable and regular enough to allow a non-unique continuation of the inner spacetime into the pathological (non-deterministic) region. Such pathological black-hole spacetimes, *if* exist, would provide a disturbing counter-example to the cosmic censorship conjecture and would signal the breakdown of determinism in general relativity.

The question of the final fate of the Penrose cosmic censorship conjecture in non-asymptotically flat spacetimes has attracted much attention from physicists and mathematicians during the last three years [4–8]. In particular, it has been proved [4–8] that the fundamental nature (singular/non-singular) of the inner Cauchy horizons in asymptotically de Sitter black-hole spacetimes is determined by a delicate interplay between two competing physical mechanisms:

- (1) The characteristic asymptotic *decay* $\psi^{\text{external}}(t \rightarrow \infty) \sim e^{-\Im\omega_0 t}$ of remnant fields in the exterior regions of the dynamically formed black-hole spacetime. Here ω_0 is the fundamental (least damped) quasinormal resonant frequency which determines the characteristic relaxation rate of the external spacetime.
- (2) The blue-shift *amplification* phenomenon $\psi^{\text{internal}}(v \rightarrow \infty) \sim e^{\kappa_- v}$ [9] which characterizes the dynamics of the infalling fields as they accumulate along the inner Cauchy horizon of the dynamically formed black hole. Here κ_- is the surface gravity of the inner black-hole horizon.

Intriguingly, the final fate of the inner Cauchy horizons inside physically realistic (dynamically formed) black holes in non-asymptotically flat spacetimes is determined by the simple dimensionless

ratio [4]

$$\Gamma \equiv \frac{\Im\omega_0}{\kappa_-} . \quad (1)$$

In particular, a dynamically formed black hole which is characterized by the dimensionless inequality $\Gamma > 1/2$ contains an inner regular Cauchy horizon that allows the corresponding spacetime to be continued in non-unique ways [4], thus violating the fundamental Penrose strong cosmic censorship conjecture.

One therefore concludes that the simple inequality

$$\Gamma \leq \frac{1}{2} \quad (2)$$

provides a necessary condition for the Einstein field equations to preserve their predictive power in non-asymptotically flat de Sitter spacetimes. It is therefore physically important to prove that the quasinormal resonant spectra of *all* dynamically formed black holes are characterized by the property [see Eqs. (1) and (2)]

$$\Im\omega_0 \leq \frac{1}{2}\kappa_- . \quad (3)$$

It is interesting to note that, using analytical techniques, it has been proved in [10] that spinning Kerr-de Sitter black-hole spacetimes respect the inequality (3) and therefore respect determinism. However, the situation is more involved in the case of charged de Sitter black holes [4–8, 11]. In particular, using analytical techniques, it has been explicitly proved in [6] that composed charged-de-Sitter-black-holes-charged-massive-fields systems in the dimensionless physical regime $\mu r_+ \ll qQ \ll (\mu r_+)^2$ [here $\{\mu, q\}$ are respectively the proper mass and charge coupling constant of the charged matter field, and $\{Q, r_+\}$ are respectively the black-hole electric charge and the radius of its event horizon] are characterized by quasinormal resonant spectra that respect the fundamental inequality (3). One therefore concludes that these dynamically formed charged black-hole spacetimes respect determinism (and, in particular, respect the Penrose strong cosmic censorship conjecture).

For most physicists, the explicit calculations presented in [6] for the validity of the necessary inequality (3) [12] in the physical regime $\mu r_+ \ll qQ \ll (\mu r_+)^2$ are certainly not enough. In particular, in order to prove the validity of the Penrose conjecture for all black-hole spacetimes, it is necessary to have a generic (that is, parameter-independent) proof that *all* physically realistic (dynamically formed) black holes are characterized by quasinormal relaxation spectra that respect the fundamental inequality (3).

In view of the complex mathematical nature of the Einstein-charged-matter field equations [4–8], it may seem that any direct attempt to obtain a general proof for the validity of the strong cosmic censorship conjecture in charged black-hole spacetimes is doomed to fail. In particular, it should be realized that a direct test of the Penrose conjecture in charged de Sitter black-hole spacetimes would require one to scan numerically the infinitely large phase space of the black-hole-field physical parameters $\{r_-, r_+, r_c, q, \mu\}$ [13] in search of a dynamically formed charged black hole that violates the necessary inequality (3) and therefore violates the strong cosmic censorship conjecture [12]. It is clear that this direct numerical approach to the problem is a truly time consuming Sisyphean task!

But we need not lose heart. Our experience in physics has taught us that the fundamental laws of nature may sometimes provide, in remarkably elegant ways, important insights about the physical properties of highly complex systems. Bekenstein’s generalized second law of thermodynamics is among these truly remarkable laws [14]. It states that the total entropy of a black-hole spacetime is a non-decreasing quantity in self-consistent quantum theories of gravity.

Intriguingly, and most importantly from the point of view of the strong cosmic censorship conjecture, it has been explicitly proved in [15] that the Bekenstein generalized second law of thermodynamics [14] implies that the characteristic relaxation time τ of a perturbed thermodynamic system is bounded from below by the remarkably compact time-times-temperature (TTT) quantum relation [15]

$$\tau \times T \geq \frac{\hbar}{\pi} . \quad (4)$$

where T is the characteristic temperature of the physical system.

Black-hole spacetimes, like mundane thermodynamic systems, are known to be characterized by a well defined temperature, which is given by the famous Bekenstein-Hawking relation [14, 16]

$$T_{\text{BH}} = \frac{\kappa_+}{2\pi} \cdot \hbar , \quad (5)$$

where κ_+ is the surface gravity of the black-hole event horizon. Substituting the Bekenstein-Hawking black-hole temperature (5) into the universal relaxation bound (4) and using the relation $\tau_{\text{relax}} \equiv 1/\Im\omega_0$ for the characteristic relaxation time of the dynamically formed black-hole spacetime, one finds that the quasinormal resonant spectra of *all* physically realistic (dynamically formed) black-hole spacetimes are characterized by the compact upper bound

$$\Im\omega_0 \leq \frac{1}{2}\kappa_+ . \quad (6)$$

We have therefore established a remarkably simple (and physically important) relation between the characteristic relaxation rates of dynamically formed black-hole spacetimes and the corresponding black-hole surface gravities.

Summary.— The Penrose strong cosmic censorship conjecture [1–3], which asserts that general relativity is a deterministic theory of gravity, has attracted much attention from physicists and mathematicians during the last five decades. In particular, the final fate of this conjecture in non-asymptotically flat charged black-hole spacetimes has been the focus of an intense debate during the last three years [4–8].

The Penrose conjecture states that the inner spacetime regions of physically realistic (dynamically formed) black holes cannot be extended in a non-deterministic (non-unique) way beyond their inner Cauchy horizons. The conjecture therefore implies that the inner horizons of black-hole spacetimes must be unstable to remnant matter fields that fall into the dynamically formed black holes. In particular, the relaxation-rate-inner-surface-gravity relation $\Im\omega_0 \leq \frac{1}{2}\kappa_-$ [see Eq. (3)] provides a necessary condition for the validity of the Penrose conjecture in asymptotically de Sitter black-hole spacetimes [4–8].

We have emphasized the fact that a direct (brute force) approach to test the validity of the Penrose strong cosmic censorship conjecture [and the closely related fundamental inequality (3)] in physically realistic black-hole spacetimes would be to scan numerically the infinitely large phase space of the quasinormal resonant spectra which characterize the late-time relaxation of the dynamically formed black holes.

Instead of following this truly Sisyphean approach, we have pointed out that the Bekenstein generalized second law of thermodynamics [14] implies that thermodynamic systems with well defined temperatures, including black holes, are characterized by the universal relaxation bound $\Im\omega_0 \leq \frac{1}{2}\kappa_+$ [see Eq. (6)] [15]. Using the characteristic inequality $\kappa_+ \leq \kappa_-$ for the black-hole surface gravities [5], one concludes that the relaxation spectra of *all* dynamically formed black-hole spacetimes are characterized by the relaxation-rate-surface-gravity inequality $\Im\omega_0 \leq \frac{1}{2}\kappa_-$. This fact implies that the corresponding inner Cauchy horizons are dynamically unstable to infalling remnant fields [4–8]. Thus, the inner black-hole spacetime cannot be extended in a non-unique (non-deterministic) way beyond these horizons.

We have therefore proved that physically realistic (dynamically formed) black-hole spacetimes *respect* the Penrose strong cosmic censorship conjecture.

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