Quantum Gravity and Dark Matter

Chiu Man Ho[1*], **Djordje Minic**[2†], **and Y. Jack Ng**[3‡]

[1]*Department of Physics and Astronomy, Vanderbilt University,*
Nashville, TN 37235, U.S.A.

[2]*Department of Physics, Virginia Tech, Blacksburg, VA 24061, U.S.A.*

[3]*Institute of Field Physics, Department of Physics and Astronomy,*
University of North Carolina, Chapel Hill, NC 27599, U.S.A.

Abstract

We propose a connection between global physics and local galactic dynamics via
quantum gravity. The salient features of cold dark matter (CDM) and modified Newto-
nian dynamics (MOND) are combined into a unified scheme by introducing the concept
of MONDian dark matter which behaves like CDM at cluster and cosmological scales
but emulates MOND at the galactic scale.

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[*]chiuman.ho@vanderbilt.edu
[†]dminic@vt.edu
[‡]yjng@physics.unc.edu

1 Introduction

The vexing problem of "missing mass" has been with us ever since Zwicky found that the individual galaxies in the Coma cluster were zipping around too fast for gravity to hold them together in a cluster. This problem started to get the attention of the scientific community after Rubin and her collaborators discovered that the clouds of hydrogen gas in several distant galaxies were orbiting the center of their galaxies at a speed far exceeding what could be accounted for by the gravitational pull due to the mass of visible baryonic matter. This mass mismatch has since been astoundingly confirmed from dwarf galaxies to galaxy groups. A simple but bold solution would be to postulate the existence of dark (invisible) matter that makes up the mass difference [1].

This apparent need for dark matter at the galactic scale is even more urgent at larger scales. Dark matter is needed to account for the correct cosmic microwave background spectrum shapes (including the alternating peaks). It is also needed to yield the correct large-scale structures and elemental abundances from big bang nucleosynthesis. (Supercomputer simulations suggest that the cosmos would look very different if dark matter did not exist.) Dark matter also provides the correct gravitational lensing. One of the most prominent examples pointing to the existence of dark matter is the Bullet Cluster, a pair of merging galaxy clusters. In this system, the gravitational lensing of background galaxies indicates that the mass is offset from the X-ray plasma, suggesting that dark matter has shifted the center of gravity elsewhere. The consensus in the cosmology community appears to be that dark matter exists! By now all this has been canonized in the concordant ΛCDM model of cosmology according to which cold dark matter accounts for about 23% of the energy and mass of the universe, 5% resides in ordinary matter, and dark energy in the form of cosmological constant has the lion's share, accounting for the remaining 72% (numerically the cosmological constant is related to the current Hubble parameter as $\Lambda \sim 3H^2$.)

But there is a fly in the ointment. At the galactic scale, dark matter can explain the observed asymptotic independence of orbital velocities on the size of the orbit only by fitting data (usually with two parameters) for individual galaxies. It can do no better in explaining the observed baryonic Tully-Fisher relation [2, 3], i.e., the asymptotic-velocity-mass ($v^4 \propto M$) relation. Another problem with dark matter is that it seems to possess too much power on small scales ($\sim 1 - 1000$ kpc) [4]. While cold dark matter works spectacularly well at the cluster and cosmic scales, it had been found to be somewhat wanting at the galactic scale.

This is in stark contrast to another paradigm that goes by the name of modified Newtonian dyanmics (MOND) [5, 6], due to Milgrom, which stipulates that the acceleration of a test mass m due to the source M is given by $a = a_N$ for $a \gg a_c$, but $a = \sqrt{a_N \, a_c}$ for $a \ll a_c$, where $a_N = GM/r^2$ is the magnitude of the usual Newtonian acceleration and the critical acceleration a_c is numerically related to the speed of light c and the Hubble scale H as $a_c \approx cH/(2\pi) \sim 10^{-8} cm/s^2$. With only a single parameter MOND can explain easily and rather successfully the observed flat galactic rotation curves and the observed Tully-Fisher

relation [7]. But there are problems with MOND at the cluster and cosmological scales. The reason may be due to the lack of a fundamental relativistic theory of MOND.

Obviously it is desirable to combine the salient successful features of both CDM and MOND into a unified scheme. This essay describes our effort in that endeavor. Succinctly, by making use of a novel quantum gravitational interpretation of (dark) matter's inertia our scheme [8] indicates that dark matter emulates MOND at the galactic scale. This work embraces a recurring theme at the current gravity research frontiers: the relationship between gravity and thermodynamics.

2   Entropic Gravity and Critical Galactic Acceleration

We start with the recent work of E. Verlinde [9, 10, 11, 12] in which the canonical Newton's laws are derived from the point of view of holography. Using the first law of thermodynamics, Verlinde proposes the concept of entropic force $F_{entropic} = T\frac{\Delta S}{\Delta x}$, where Δx denotes an infinitesimal spatial displacement of a particle with mass m from the heat bath with temperature T. He then invokes Bekenstein's original arguments concerning the entropy S of black holes [13] by imposing $\Delta S = 2\pi k_B \frac{mc}{\hbar}\Delta x$. Using the famous formula for the Unruh temperature, $k_B T = \frac{\hbar a}{2\pi c}$, associated with a uniformly accelerating (Rindler) observer [14, 15], he obtains $F_{entropic} = T\nabla_x S = ma$, Newton's second law (with the vectorial form $\vec{F} = m\vec{a}$, being dictated by the gradient of the entropy).

Next, Verlinde considers an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T. Then, he assumes the equipartition of energy $E = \frac{1}{2}Nk_B T$ with N being the total number of degrees of freedom (bits) on the screen given by $N = Ac^3/(G\hbar)$. Using the Unruh temperature formula and the fact that $E = Mc^2$, he obtains $2\pi k_B T = GM/r^2$ and recovers exactly the non-relativistic Newton's law of gravity, namely $a = GM/r^2$. But this is precisely the fundamental relation that Milgrom is proposing to modify so as to fit the galactic rotation curves. Therefore it is now natural to ask whether there is an entropic [13] or holographic [16, 17, 18, 19] interpretation behind Milgrom's modification of Newton's second law.

We first have to recognize that we live in an accelerating universe (in accordance with the ΛCDM model). This suggests that we will need a generalization [8] of Verlinde's proposal [9] to de Sitter space with a positive cosmological constant (which, we recall, is related to the Hubble parameter H by $\Lambda \sim 3H^2$). The Unruh-Hawking temperature as measured by a non-inertial observer with acceleration a in the de Sitter space is given by $\sqrt{a^2 + a_0^2}/(2\pi k_B)$ [20, 21], where $a_0 = \sqrt{\Lambda/3}$ [16]. Consequently, we can define the net temperature measured by the non-inertial observer (relative to the inertial observer) to be $\tilde{T} = [(a^2 + a_0^2)^{1/2} - a_0]/(2\pi k_B)$.

We can now follow Verlinde's approach.[4] Then the entropic force, acting on the test mass m with acceleration a in de Sitter space, is given by $F_{entropic} = \tilde{T}\,\nabla_x S = m[(a^2 + a_0^2)^{1/2} - a_0]$. For $a \gg a_0$, the entropic force is given by $F_{entropic} \approx ma$, which gives $a = a_N$ for a test mass m due to the source M. But for $a \ll a_0$, we have $F_{entropic} \approx ma^2/(2a_0)$ which, upon equated with ma_N, yields $a \approx \sqrt{2a_N a_0}$. Thus we have derived Milgrom's scaling or MOND with the correct order of magnitude for the (observed) critical galactic acceleration $a_c = 2a_0 \sim \sqrt{\Lambda/3} \sim H \sim 10^{-8} cm/s^2!$ [5] From our perspective, MOND *is a (successful) phenomenological consequence of quantum gravity.*

Having derived MOND we can now write the entropic force, in the regime $a \ll a_0$, as $F_{entropic} \approx m\frac{a^2}{2\,a_0} = F_{Milgrom} \approx m\sqrt{a_N a_c}$ implying that $a = (\,4\,a_N\,a_0^2\,a_c\,)^{\frac{1}{4}} = (\,2\,a_N\,a_0^3/\pi\,)^{\frac{1}{4}}$. Numerically, it turns out that $2\pi a_c \approx a_0$, and so we set $a_c = a_0/(2\pi)$ for simplicity. The flat galactic rotation curves as well as the Tully-Fisher relation can be easily recovered by using $F_{centripetal} = m\frac{a^2}{2\,a_0} = \frac{m\,v^2}{r}$ to solve for the terminal velocity v.

3   MONDian Dark Matter

To see how dark matter can behave like MOND at the galactic scale, let us continue to follow Verlinde's holographic approach. Invoking the imaginary holographic screen of radius r, we can write[6] $2\pi k_B \tilde{T} = \frac{G\,\tilde{M}}{r^2}$, where $\tilde{M}$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$. But, as we will show below, consistency with the discussion in the previous section (and with observational data) demands that $\tilde{M} = M + M'$ where M' is some unknown mass — that is, dark matter. Thus, we need the concept of dark matter for consistency.

First note that it is natural to write the entropic force $F_{entropic} = m[(a^2 + a_0^2)^{1/2} - a_0]$ as $F_{entropic} = m\,a_N[1 + (a_0/a)^2/\pi]$ since the latter expression is arguably the simplest interpolating formula for $F_{entropic}$ that satisfies the two requirements: $a \approx (2a_N a_0^3/\pi)^{1/4}$ in the small acceleration $a \ll a_0$ regime, and $a = a_N$ in the $a \gg a_0$ regime. But we can also write F in another, yet equivalent, form: $F_{entropic} = mG(M + M')/r^2$. These two forms of F illustrate the idea of CDM-MOND duality [8]. The first form can be interpreted to mean that there is no dark matter, but that the law of gravity is modified, while the second form means that there is dark matter (which, by construction, is consistent with MOND) but that the law of gravity is not modified. The second form gives us a very intriguing dark matter profile: $M' = \frac{1}{\pi} \left(\frac{a_0}{a}\right)^2 M$. Dark matter of this kind can behave as if there is no dark matter but MOND. Therefore, we call it "MONDian dark matter" [8]. One can solve for M' as a

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[4]We only need to replace the T in Verlinde's argument by $\tilde{T}$ for the Unruh temperature.

[5]Actually Milgrom did observe [22] that the generalized Unruh temperature $\tilde{T}$ can give the correct behaviors of the interpolating function between the usual Newtonian acceleration and his suggested MONDian deformation for very small accelerations. He was right; but, unlike us, he could offer no justification.

[6]We need to replace the T and M in Verlinde's argument by $\tilde{T}$ and $\tilde{M}$ respectively.

function of r in the two acceleration regimes: $M' \approx 0$ for $a \gg a_0$, and (with $a_0 \sim \sqrt{\Lambda}$)

$$M' \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$$

for $a \ll a_0$. Intriguingly *the dark matter profile* we have obtained *relates, at the galactic scale,*[7] *dark matter (M'), dark energy (Λ) and ordinary matter (M) to one another.* As a side remark, this dark matter profile can be used to recover the observed flat rotation curves and the Tully-Fisher relation.

4   Inertia in Quantum Gravity

As the example of MOND illustrates, the inertia – the response of a body to force – is not an inherent property of bodies. It depends on the background medium. Let us discuss this phenomenon in a more general and wider context. Adopting the thermo-field-dynamics language of Israel [23] and others [24, 25], we start from the familiar expansion of the Minkowski vacuum ψ in terms of an ensemble sum over the entangled left and right Rindler vacuum ϕ: $\psi_M = \sum_i e^{-\beta E_i} \phi_i^L \times \phi_i^R$, where β is the inverse of the Unruh temperature $T \sim \hbar a/c$. Better yet, instead of $\phi^{L,R}$ we can use, in the above formula, the exact quantum gravity states, such as fuzzball states introduced by Mathur [26], the presumed exact no-horizon states of string theory (viewed as a quantum theory of gravity).

Replacing the Minkowski vacuum with the global de Sitter vacuum and the entangled left and right Rindler patches with the entangled left and right static patches of de Sitter (or better still, with their fuzzball counterparts) we can expand the de Sitter vacuum with cosmological constant Λ as

$$\Psi_{dS}^\Lambda = \sum_i e^{-\beta_{dS} E_i} \Phi_i^L \times \Phi_i^R.$$

This formula is in the spirit of the discussion of holography in de Sitter space as presented in [27]. Here the Unruh temperature in the de Sitter space is given by $\tilde{T} = [(a^2 + a_0^2)^{1/2} - a_0]/(2\pi k_B)$ with $a_0 = \sqrt{\Lambda/3}$ introduced above. The weighting factors in the ensemble sum depend on the energies $E_i \sim m_i c^2$ with m_i being the invariant masses associated with the exact quantum gravity spectrum (which we do not know how to calculate at present). Thus, for the Minkowski case, we have ratios like $m_i a^{-1}$ in the ensemble weights (in the units of $c = 1, \hbar = 1$ and $k_B = 1$). But, for the de Sitter case, we obtain these ratios only in the limit of $a \gg a_0$, whereas in the limit of $a \ll a_0$ the ratios are $m_i(a^2/a_0)^{-1}$, where a_0 is the critical acceleration a la Milgrom. The latter result for the de Sitter case can be reinterpreted to mean that *the inertial properties of massive particles* with mass m_i *have become acceleration- and Λ-dependent*! (This is analogous to what happens to the inertial properties of particles

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[7]One may wonder why MOND works at the galactic scale, but not at the cluster or cosmic scale. One of reasons is that, for the larger scales, one has to use Einstein's equations with non-negligible contributions from the pressure and explicitly the cosmological constant, which have not been taken into account in the MOND scheme [8].

in special relativity, where the dependence is on the velocity of the particle and c, the speed of light.)

Therefore, the weights of the exact formula which represents the expansion of the exact de Sitter quantum gravity state in terms of entangled exact stationary quantum gravity states, such as fuzzballs, associated with the left and right static patches, are scale-dependent, and the weights determine what we mean by the mass at different scales. This general argument suggests that Milgrom's scaling is just a new physical phenomenon in which the inertial properties are acceleration- and Λ-dependent.

5   Discussion

To recapitulate, we have given an entropic and holographic dual description of the Milgrom scaling associated with galactic rotation curves and the Tully-Fisher relation by showing how dark matter can emulate the modified Newtonian dynamics at the galactic scale. In addition we have argued that the inertia of matter is quantum gravitational and is deeply connected to dark energy.

The last point reminds us of the relation between nuclear physics and the theory of quantum chromodynamics (QCD). We can make the following analogy: the phenomenological lagrangians of nuclear physics would correspond to the phenomenological treatments of matter in particle physics. But the true picture should be found in quantum gravity, the analog of QCD in this comparison. Furthermore, the Milgrom scaling can perhaps be viewed as an analog of the Bjorken scaling and quantum gravity corrections to the Unruh formula as the analog of the logarithmic corrections to the Bjorken scaling as provided by asymptotic freedom in QCD.

We conclude with some open questions to be investigated. Our arguments have been mainly thermodynamical, hence the precise nature of MONDian dark matter is still unclear; we should construct a microscopic theory which would also address the question of crossover between the Newtonian and MOND regimes. The dark matter profile we have obtained (at the galactic scale) hints at a fixed energy density ratio between the three different cosmological components of the Universe; how does this relation help to alleviate the coincidence problem? That same relation seems to suggest that the microscopic MONDian dark matter degrees of freedom may have knowledge of the non-local cosmological constant; what are some of the phenomenological implications (for example, in dark matter searches)? Because we have the exact formula for the Unruh acceleration, we can now look at the higher order terms (in powers or inverse powers of a^2/Λ) in the expansion of the relevant expressions to examine possible phenomenological consequences. For example, are there corrections to the Milgrom scaling? Would the corrections improve MOND's agreement with the rotation curves? What are the effects on the Tully-Fisher relation? Are the correction terms relevant to sub-galactic scales? Finally we have given a general argument showing that inertia of a

body depends on the physical vacuum. Can we derive the entropic force directly in this general framework? And what novel properties of dark matter and distinctive phenomenologies can be uncovered?

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