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Einstein's Dream

Richard T. Hammond
rhammond@email.unc.edu
Department of Physics and Astronomy
University of North Carolina at Chapel Hill
Chapel Hill, North Carolina and
Army Research Office
Research Triangle Park, North Carolina
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It is shown the antisymmetric part of the metric tensor is the potential for the torsion field, which arises from intrinsic spin. To maintain gauge invariance, the nonsymmetric part of the metric tensor must be generalized to include the electromagnetic field. This result leads to a link between the cosmological constant and the electromagnetic field.

Soon after Einstein developed the theory of general relativity (GR) he pursued a course of research that would haunt him for the rest of his life. GR was based on Riemannian geometry but, with the work of Levi-Civita, Ricci and others, a much more general geometry was developed, and there was no longer any reason to assume the metric tensor was symmetric. With the generalization of Minkowski geometry to Riemannian geometry, new physics was discovered–gravity. The generalization to non-Riemannian geometry launched a new pursuit, and the search for the physical interpretation of the non-symmetric part of the metric tensor (NMT) was on.

Despite many failures, Einstein had a strong belief the NMT was related to the electromagnetic field. However, as the years went by, there were indications the NMT was related to something other than electromagnetism. The first hint came from Papapetrou,[1] who was motivated by the Einstein Strauss field theory.[2] Papapetrou reasoned, since the metric tensor is not symmetric the energy momentum tensor cannot be symmetric. He then showed that the antisymmetric part of an energy momentum tensor is related to intrinsic spin, not electromagnetism.

A few years later Sciama[3] proposed the NMT was related to spin, and gave general arguments why this was so. Soon, however, he dropped this idea and assumed spin was associated with torsion. A little latter Moffat[4] assumed the NMT was just another part of gravity, the electroweak theory was established showing electromagnetism was associated with the weak force, and by then Einstein's dream of a unified theory of gravity and electromagnetism was dead and buried.

However, there is no reason why the NMT should be zero. The issue about expanding about a background space has been settled[5] and the main force dissuading further research in this area is the effluvium of failure.

In this essay we embark on a search for the physical significance of the NMT, leaving behind as many prejudices and assumptions as we can. So, we follow the highly successful program of Einstein's original theory as closely as possible, with the exception we relax the symmetry assumption on the metric tensor, which naturally leads to a non-symmetric affine connection as well. We consider variations with respect to the metric tensor and it derivatives.

To formulate Einstein's GR one other assumption is used: The covariant derivative of the metric tensor is zero. This insures the length of a vector upon parallel transport is unchanged. However, this notion makes little sense for the NMT. This is because we know the equation of motion for a particle is generally not a geodesic when the metric tensor and/or the affine connection is not symmetric. Thus, in general, we have $\nabla_\sigma g_{\mu\nu} = Q_{\sigma\mu\nu}$, i.e., the covariant derivative of the metric tensor is equal to the non-metricity tensor. But what is $Q_{\sigma\mu\nu}$?

To determine the non-metricity tensor we assume the theory reduces to GR in the limit the metric tensor is symmetric. This implies as the non-symmetric part of the metric tensor goes to zero, the non-symmetric part of the affine connection goes to zero.

Thus, there are three basic assumptions used to formulate the theory (aside from choosing the matter terms and assuming variations vanish on the boundary): 1) the curvature scalar plus the cosmological constant, times the square root of the determinant, is the Lagrangian density, 2) variations are taken with respect to the metric tensor and its derivatives, and 3) as the antisymmetric part of the metric tensor vanishes, the equations reduce to those of the original 1915 GR.

With so few, yet natural, assumptions we hope to have a better chance of finding out the true physical interpretation of the non-symmetric part of the metric tensor. We say above there is evidence that it may relate to spin, but that in no way guided the choice of the three assumptions. Nature is hard to understand and complicated, the less we assume at the outset the less likely we are go astray.

Assumptions 1 and 2 are self-evident, but assumption 3 is crucial to the result. This happens as follows; we assume the non-metricity condition given above and write $g_{\mu\nu} = \gamma_{\mu\nu} + \phi_{\mu\nu}$, which represent the symmetric and antisymmetric parts of the metric tensor. Then, by permuting the indices and adding these equations, it is found derivative terms of $\phi_{\mu\nu}$ equal terms involving the metric tensor and the non-metricity tensor. In order to make the torsion go to zero as $\phi_{\mu\nu}$ goes to zero, the non-metricity tensor must be twice the torsion tensor, details may be found in the literature.[7] This is satisfactory since it reduces the GR in the absence of torsion.

The field equations are then derived by performing variations with respect to the metric tensor. When the field equations are linearized (with respect to $\gamma_{\mu\nu}$ a remarkable thing happens. They reduce to a theory of gravitation with a symmetric metric and torsion potential given by $\phi_{\mu\nu}$. It is gravity with string theory type torsion and has been well studied.[6] One of the most important results of those investigations is this, the correct conservation law for total angular and spin is derived if the torsion exists and the source of torsion is intrinsic spin.

This is an extremely valuable result. Not only does it finally give us definitive interpretation of the antisymmetric part of the metric tensor, many other checks have been performed, namely, coupling to the Dirac equation, coupling to strings for the matter action, the power of torsion waves due to spin coupling was found, and others.[6] In addition, verifying Papapetrou's work, it was shown how the intrinsic spin of the canonical electromagnetic field gives rise to antisymmetric metric tensor.

There is one problem. In the symmetric theory just referred to, and in string theory, torsion enjoys gauge invariance under $\phi_{\mu\nu} \to \phi_{\mu\nu} + \xi_{\mu,\nu} - \xi_{\nu,\mu}$. Since the square root of the determinant appears in the Lagrangian density, this gauge invariance is lost.

The same thing happens in string theory when a point charge is put on a brane, the torsional gauge invariance is lost. The resurrect it, a new vector field is added to the torsion potential of the form $\mathcal{F}_{\mu\nu} \equiv \lambda_{\nu,\mu} - \lambda_{\mu,\nu}$ such that when $\phi_{\mu\nu} \to \phi_{\mu\nu} + \xi_{\nu,\mu} - \xi_{\mu,\nu}$, $\lambda_\nu \to \lambda_\nu - \xi_\nu$. The same procedure is adopted here by letting $\phi_{\mu\nu}$ be replaced by $\phi_{\mu\nu} + \lambda_{\nu,\mu} - \lambda_{\mu,\nu}$.

This completes the theory and gauge invariance has been achieved. From its definition $\mathcal{F}_{\mu\nu}$ looks to be proportional to electromagnetic field. For that to be the case, there would have to be the kinetic term for electromagnetism in the Lagrangian density. The remarkable fact is, *it is already there.* To see this, consider the determinants of $g_{\mu\nu}$, $\gamma_{\mu\nu}$, and $\phi_{\mu\nu}$, which are g, γ, and ϕ. Then we have

$$g = \gamma + \phi + \frac{\gamma}{2}\gamma^{\alpha\beta}\gamma^{\mu\nu}\phi_{\alpha\mu}\phi_{\beta\nu}. \tag{1}$$

With this we have the extraordinary result

$$\int \sqrt{-g}d^4x(R+\Lambda) \rightarrow \tag{2}$$
$$\int \sqrt{-\gamma}d^4x\left({}^oR - S^{\alpha\beta\sigma}S_{\alpha\beta\sigma} + \frac{1}{16\pi}F^{\alpha\beta}F_{\alpha\beta} + \Lambda\right) + ...$$

where ${}^oR$ is the curvature scalar of Riemannian geometry, $F_{\mu\nu} \equiv \kappa(\xi_{\nu,\mu} - \xi_{\mu,\nu})$ and κ is taken to be given by

$$\kappa = G/c^2\sqrt{\Lambda}. \tag{3}$$

Thus, after recognizing the antisymmetric part of the metric tensor is the potential of the spin field, torsion, we found in order to have gauge invariance the metric tensor must be modified by adding the electromagnetic field tensor to it. With the presence of the cosmological constant, we found the surprising result that the kinetic term for the electromagnetic field is already present. The existence of the cosmological constant has long been a mystery, and the most recent observations show it is not zero. But from the above, we find the cosmological constant must exist.

Theories with non-zero torsion and a symmetric tensor are sometimes said to be ad hoc, the argument being there is no need for torsion. But with a non-symmetric metric tensor the torsion is not zero, and arises naturally. And it is necessary, since without it the correct angular momentum conservation law is not obtained.

This formulation solves another age old problem, it naturally couples the electromagnetic field to torsion. On the right side of (2) the dots include terms in which the electromagnetic field is multiplied by torsion. This may lead, finally, to new tests of torsion because electromagnetic fields with intensities $10^{21}\mathrm{Wcm}^{-2}$ have been produced, and with ELI coming on board it is expected to reach intensities 1,000 time higher. If polarized, the spin density is between million and a billion times the spin density in matter.

There are no new unknown constants, which is important because the theory is testable. The dimensionless constant κ was fixed by (3), so the only undetermined constant is the cosmological constant.

Precisely 100 years after the famous eclipse that confirmed GR, has Einstein's dream of a unified theory been realized? Not exactly, the spin field is the key, and must be present, the electromagnetic field is a necessary ingredient to have gauge invariance, but the electromagnetic field is there after all.

1490 words

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