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The necessity of torsion in gravity

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Abstract

It is shown that torsion is required for a complete theory of graviation, and that without it, the equations of gravitation violate fundamental laws. In the first case, we are reminded that, in the absence of external forces, the correct conservation law of total angular momentum arises only if torsion, whose origin is intrinsic spin, is included into gravitation. The second case considers the "mass reversal" transformation. It has been known that under a global chiral transformation and "mass to negative mass" transformation, the Dirac equation is invariant. But global transformations violate special relativity, so this transformation must be made local. It is shown that the torsion is the gauge field for this local invariance.

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1 Classical view

Torsion of spacetime has strong theoretical roots. The local Poincare gauge theory, a natural extension of global Poincare invariance of particle physics, requires the rotational gauge potential–torsion.[1] From string theory, torsion, as well as a scaler field, is required, and it has been shown how this theory yields the correct equation of motion.[2] In both of these approaches the torsion arises from intrinsic spin, and just as mass and charge give rise to a field, so does torsion.

Despite the theoretical motivation for torsion, there is sometimes an argument that torsion is *not required*. The point of this essay is to show that torsion *is required*, and I will present two arguments. One is already known, but it may not be widely known. This rests upon the conservation of angular momentum, but first, let me write down the equations.

Torsion is assumed to be derived from a potential according to,

$$S_{\mu\nu\sigma} = \psi_{[\mu\nu,\sigma]}. \tag{1}$$

The gravitational field equations are

$$G^{(\mu\nu)} - 2S^{\mu}{}_{\alpha\beta}S^{\nu\alpha\beta} = kT^{\mu\nu} \tag{2}$$

and the torsional field equations are

$$S^{\mu\nu\sigma}{}_{;\sigma} = -kj^{\mu\nu}. \tag{3}$$

It is known that the equations of motion of a particle follow from the field equations. This is extremely important, since any putative changes to general relativity can be put to the test of reality. Torsion is a case in point, and the equations of motion have been derived. The rotational equation of motion was also derived and it was shown that, in the absence of external forces

$$\frac{d}{d\tau}J^{\mu\nu} = 0 \tag{4}$$

where

$$J^{\mu\nu} = L^{\mu\nu} + S^{\mu\nu} \tag{5}$$

where $L^{\mu\nu}$ is the orbital angular momentum and $S^{\mu\nu}$ is the intrinsic spin, which gives rise to the torsion field. Torsion is necessary to derive (4), which we know to be the correct law of conservation of total angular momentum. The $S^{\mu\nu}$ cannot be accounted for by any motions of matter, and is the source of torsion. Thus, if torsion vanishes, then we have $\frac{d}{d\tau}L^{\mu\nu} = 0$ –the wrong physical result. In order to obtain the correct result, *we must include torsion into gravitation.* All of these results have been collected in a review article.[2]

2 Quantum View

In the above, the source of torsion, the intrinsic spin, was modeled by a phenomenological source, either by using an intrinsic vector or by using a string as the source. Thus, a string, due to its shape, gives rise to spin. However, quantum mechanically, we have

$$\gamma^a D_a \psi + \frac{imc}{\hbar}\psi = 0, \tag{6}$$

and

$$S^{\alpha\beta\sigma}{}_{;\sigma} = -\frac{i\hbar ck}{2}\Xi^{\alpha\beta\sigma}{}_{;\sigma} \tag{7}$$

where now the source is given by $\Xi^{\alpha\beta\sigma} = (1/2)\overline{\psi}\gamma^{[\alpha}\gamma^{\beta}\gamma^{\sigma]}\psi$,
and the covariant derivative is

$$D_a\psi = \psi_{,a} - \frac{1}{4}\Gamma_{abc}\gamma^b\gamma^c\psi \tag{8}$$

where

$$\Gamma_{abc} = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab} + S_{abc}. \tag{9}$$

The Γ_{abc}, without torsion, may be viewed as the gauge field for the group of diffeomorphisms, or the local Lorentz group. Now we shall show that the S_{abc} field is *required* when we make a global symmetry of the Dirac equation local. This puts the torsion field on par with fields generated by making the global $U(1)$ local, or the local $SU(3)$ symmetry. In the following we will ignore the effect of the gravitational field.

The global symmetry of the Dirac equation is the combined transformations

$$\psi \rightarrow e^{i\alpha\gamma^5} \tag{10}$$

($\alpha =$ constant), and

$$m \rightarrow -m \tag{11}$$

This global transformation, called mass reversal by Tiomno, has been known for some time,[3] (in these references the authors were trying to model the electromagnetic and weak interactions) but as we know, global symmetries violate special relativity, so this must be made local.

To make the transformation local we let $\alpha \rightarrow \alpha(x)$. Now, although the torsion is derived from a potential and is invariant under the associated gauge transformation, it enters the Dirac equation, through the covariant derivative, with the field, not the potential. However, there is another invariance transformation that arises from (3): It is $S_{\alpha\beta\sigma} \rightarrow S_{\alpha\beta\sigma} + \epsilon_{\alpha\beta\sigma\nu}\xi^{,\nu}$. One sees that (3) remains unchanged. In fact, it has been shown that a scaler field of the form $\epsilon_{\alpha\beta\sigma\nu}\xi^{,\nu}$ must be added to the torsion when (3) is integrated.[2]

Thus, the equation

$$\gamma^a \partial_a \psi - \frac{1}{4} S_{abc} \gamma^a \gamma^b \gamma^c \psi + \frac{imc}{\hbar}\psi = 0 \tag{12}$$

is invariant under the simultaneous local chiral transformation

$$\psi \rightarrow e^{i\alpha(x)\gamma^5} \tag{13}$$

along with

$$m \rightarrow -m \tag{14}$$

$$S_{\alpha\beta\sigma} \rightarrow S_{\alpha\beta\sigma} + \epsilon_{\alpha\beta\sigma\nu}\xi^{,\nu}, \tag{15}$$

provided,

$$\alpha = -\frac{3}{2}\xi. \tag{16}$$

With gravitation, it may be shown that the Lagrangian density, R, is not invariant under (15). However, it has been shown that under (15) the equations of motion are unchanged.[6] Thus, we take this as an exact symmetry in Minkowski spacetime with torsion, but broken by gravitation.

It is obvious that we may actually deduce the existence of torsion field by proceeding in the opposite direction, i.e., starting with the Dirac equation in Minkowski space, and introducing a covariant derivative that makes the global invariance stated above local.

In summary I would like to say that torsion is an absolutely necessary ingredient of gravitation. It has already been shown that the correct equation for conservation of total angular momentum can only be achieved with torsion that arises from intrinsic spin, but now there is another proof. It rests on the notion of making a global transformation, that violates the spirit of special relativity, into a local gauge theory. This result lays the foundation for considerable future research and new ways to possibly measure torsion.[5]

References

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