

# Emergence of Quantum Field Theory in Causal Diamonds

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## Abstract

The experimental successes of quantum field theory do not justify using it to describe even a finite fraction of the entanglement entropy of a causal diamond with its exterior, in the limit of large diamonds. Susskind and Uglum and Jacobson [1] conjectured that this divergent entropy could be thought of as a renormalization of Newton's constant in the Bekenstein-Hawking formula, *if we applied that formula to arbitrary causal diamonds*. Jacobson [34] showed that this leads to a derivation of the null projection of Einstein's equations as the hydrodynamic equations of the area law for arbitrary diamonds, a derivation which has the added virtue of demonstrating that the cosmological constant is not an energy density. Using a gauge choice [3] adapted to causal diamond boundaries, we revisit arguments of Carlip and Solodukhin [4] [5] that the proper theory of near horizon states is a (cut-off) 1 + 1 dimensional conformal field theory, with central charge proportional to the transverse area. This leads [6] to a universal formula for fluctuations of the modular Hamiltonian of a diamond, which we argue is compatible with the explanation [33] [8] [30] of the temperature of de Sitter space in terms of an identification between localized energy and the number of constrained q-bits of the holographic degrees of freedom.

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# 1 Causal Diamonds, Field Theory and Experiment

The justly advertised agreement between QFT and experiment is all based on experimental apparatus traveling on time-like trajectories that are close to the geodesic in a causal diamond in Lorentzian space-time. A causal diamond is the set of points traced out by all time-like trajectories between a given pair of time-like separated points, the geodesic being the trajectory of maximal proper time. The boundary of the diamond is a null surface and the maximal area  $d - 2$  surface on the boundary is the diamond's *holoscreen*.

In a remarkable paper [9], the authors showed that the experimental success of QFT was unaffected if one restricted the Hilbert space to a space of dimension  $e^{S_{BH}^{\frac{d-1}{d}}}$  of states localized around the trajectory on which the experimental apparatus resides.  $S_{BH}$  is the Bekenstein-Hawking entropy of the holoscreen of the diamond. With the cutoff used by [9] possible violations of QFT lay just outside the bounds of current experiments, but alternate cutoff schemes proposed in [10] put them out of reach for the foreseeable future.

The significance of these results is that the success of QFT does not test its validity as a description of very short wavelength states propagating on nearly null trajectories near the boundary. We can define an "energy"  $H = L_0 L_P^{-1}$  associated with the geodesic in the causal diamond, which is the generator of some diffeomorphism, which leaves the diamond invariant, and whose flow lines are time-like within the diamond and space-like outside it. We also insist that the diamond's geodesic be one of the flow lines, and normalize the generator so that it measures proper time on its flow lines. For space-times conformal to maximally symmetric spaces, these generators have been worked out in [11] [12]. In dS space, the generator  $H$  approaches the static time translation as proper time tends to infinity. With this definition of energy, short wavelength states propagating near the horizon have low energy, and in fact, in QFT, there are an infinite number of states with arbitrarily low energy. If we cut off the wavelengths at around  $L_P$ , then the states typically have energy of order  $1/R$ .

We can gain some more insight if we consider the case of Minkowski space [11] and do a conformal transformation. Then the interior of a causal diamond maps into a Rindler wedge and  $L_0$  maps into the Unruh [13] boost operator that leaves the wedge invariant. Our choice of  $L_P^{-1}$  as the energy scale corresponds to insisting on Planck scale acceleration for the Unruh detector. We're now brought back to the old problem of calculating black hole entropy in field theory. Of course, it comes out infinite, because of very short wavelength states that have very low asymptotic energy. One can try to cut this off with 't Hooft's brick wall [14] and pretend that one understands it, but the correct attitude is better expressed by the proposals of [1] for dealing with the leading UV divergence of the entanglement entropy of a diamond in quantum field theory [15] as a renormalization of  $G_N$  in the BH area law. That is to say, it was an admission that the correct physical description of near horizon states was to be found in an as yet to be constructed theory of quantum gravity, whose basic principle was the area law for entropy.

The brick wall proposal and attempts to understand black hole entropy in QFT lead directly to the firewall paradox [16]. They also fail to explain a very simple feature of the physics of black holes with negative specific heat, which leads to an equally simple resolution of the firewall paradox. Consider a state in which an "elementary" particle of mass  $m$  is dropped into a black hole of mass  $M$  from a position just above the horizon, as a state in

the Hilbert space describing the final black hole that's formed by the infall. The final state has an entropy excess

$$\Delta S = mL_P(R/L_P)^{(d-3)+\frac{(d-4)}{(d-3)}}. \quad (1.1)$$

This is large as long as  $m \gg R^{-1}$  and this entropy excess has no explanation in QFT. It must refer to a large set of q-bits that are frozen in the initial state, and are somehow responsible for the interactions between the particle and the black hole. In [17] it was suggested that matrix models provided an example of systems where this sort of freezing of degrees of freedom could put off equilibration of subsystems for an extended period. In order to explain the absence of a firewall, one must assume that the proper time along the infalling detector's geodesic can be mapped onto that of an orbiting detector a few Schwarzschild radii away from the black hole. This is related to the phenomenon of "mirages" on the black hole horizon. For example, the orbiting detector sees the meeting of two oppositely charged particles behind the black hole horizon as the closing of a dipole on the horizon.

The purpose of the present essay is not to recapitulate arguments about the firewall paradox, but to point out that the field theoretic description of near horizon states has no basis in experimental fact and can lead to theoretical errors. It has indeed led to an error in recent descriptions of the density matrix of dS space [27]

## 2 The Density Matrix of de Sitter Space

In the late 1990s, after the discovery of the acceleration of the universe, a number of researchers began to think seriously about the implications of an asymptotically dS universe. This led Fischler, Susskind and Bousso [32] to propose the Covariant Entropy Bound. We only realized much later that Ted Jacobson had shown [34] that assuming this bound was saturated for any causal diamond led to a derivation of the equations

$$k^\mu k^\nu (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 8\pi G_N T_{\mu\nu}) = 0. \quad (2.1)$$

as the hydrodynamic equations of the *Covariant Entropy Principle*: the conjecture that the modular Hamiltonian of a causal diamond satisfies  $\langle K \rangle = \frac{A}{4G_N}$ . This is a validation of the CEP and simultaneously shows that the cosmological constant should NOT be thought of as a local contribution to the energy density, a conclusion supported by the AdS/CFT correspondence, by the CKN argument, and (if one believes it) by the Banks-Fischler conjecture [7] relating the radius of dS space to the dimension of the Hilbert space.

Given these conjectures, Fischler and I were confronted by the question of how to explain the temperature of dS space. The Schwarzschild dS entropy formula suggested the idea that objects localized on a static geodesic, were constrained states of the holographic degrees of freedom, but it wasn't until a visit to Santa Cruz by a young post-doc, Tomeu Fiol, that this idea gelled. Fiol showed me a simple formula involving fermionic degrees of freedom organized in a matrix, which reproduced the dS black hole entropy formula in 4 dimensions [8].

This explanation of dS temperature appears to assume that the density matrix of dS space is maximally uncertain. At the time, Fischler and I were content to assume this, since we were looking for a relation, the CEP, that held for arbitrary causal diamonds in arbitrary space-times with no Killing vectors, and no thermal states. Raphael Bousso [32]

concluded independently that the CEP referred to the dimension of the Hilbert space of a diamond. Fischler and I did not however confront the contradiction with another strand of our research into the quantum theory of the very earliest moments of cosmology. Fischler and Susskind [32] had argued that the equation of state in the very early universe would violate the CEB unless it was  $p = \rho$ . After some false starts, we realized that a mathematical model that reproduced this equation of state was a 1 + 1 dimensional CFT living on a longitudinal interval surrounding the apparent horizon, with a central charge that grows with the area of that horizon. Much later, we realized that if one followed this by a period in which the central charge did not change, one had the beginnings of a holographic model of inflation [31]. *This assumes that the modular Hamiltonian of dS space is the  $L_0$  generator of a 1 + 1 CFT with central charge proportional to the area of the horizon in Planck units, not a constant.* At the time we did not know of the prior work of Carlip [4] who had proposed just such a description for the horizon of a general black hole. I did not learn about the even more transparent presentation of these results by Solodukhin [5] until Kathryn Zurek pointed it out in the course of our joint work on fluctuations in causal diamonds [6].

Let me state the results of that work in the most transparent way possible. If one looks at the near horizon region of a generic causal diamond, whose size is large enough that the Einstein-Hilbert action is a good approximation to gravitational dynamics, the metric factorizes into a nearly Minkowski two dimensional metric and a transverse metric whose fluctuations are small. This is most easily seen in a gauge invented by the Verlinde brothers [3] to understand 't Hooft's [18] Commutation Relations between light ray operators. In the Verlinde gauge, the metric is exactly the direct sum of a two dimensional Lorentzian metric  $g_{ab}$  and a  $d - 2$  dimensional Riemannian metric  $h_{mn}$ . One can take the near horizon limit in any causal diamond by scaling  $g \sim L_P^2$ ,  $h \sim L^2$ , with  $L \gg L_P$ . In terms of the rescaled metrics, the Einstein-Hilbert action becomes

$$S_{EH} = (L/L_P)^{d-4} S_{top} + (L/L_P)^{d-2} S_{\perp}. \quad (2.2)$$

In [3] the authors show that  $S_{top}$  is exactly soluble and leads to the 't Hooft commutation relations. The analysis of Carlip and Solodukhin shows that  $S_{perp}$ , which can clearly be treated semi-classically, simplifies even further in the near horizon limit.

In the language used by Solodukhin, fluctuations of the conformal factor of the transverse metric are encoded in a two dimensional Liouville field with central charge proportional to the area of the background  $h_0$ . All other terms in the action are "classically irrelevant perturbations" of this CFT in the near horizon limit. The basic hypothesis of Carlip and Solodukhin, generalized by [6] from black holes to arbitrary causal diamonds, is that in the quantum theory of gravity the Virasoro algebra of this classical CFT is realized by a quantum CFT with an  $L_0$  generator that is bounded from below, so that Cardy's theorem [19] applies. This leads to a correct, state counting, computation of the entropy of essentially all black holes, and to the universal formula for fluctuations of the modular Hamiltonian

$$\langle (K - \langle K \rangle)^2 \rangle = \langle K \rangle. \quad (2.3)$$

Several recent papers [26] [28] [27] have revived the conjecture [7] [8] that the entanglement spectrum of the density matrix of dS space is flat. All of them work in the infinite entropy limit. The first, which applies only to  $dS_3$ , is based on a saddle point approximation

to a TTbar deformation of a  $1 + 1$  dimensional CFT. It seems plausible that the fluctuation corrections to that model will be proportional to the expectation value of  $K^1$ . The second is the infinite temperature limit of the double scaled SYK model. It's not clear how the temperature is supposed to be scaled when the entropy is finite. The third proposal is the one most directly connected to the issues raised here. These authors argue that one needs to assume a flat entanglement spectrum if one wants to predict a thermal spectrum not just for low probability events in which localized objects spontaneously appear near the geodesic in the static patch of dS space, but also for states with static energy  $\sim 1/R$ , many of which are experimentally inaccessible to a static detector. The question posed by [9] of just what experiment tells us about the validity of QFT in a causal diamond is directly relevant to the assumptions made in [27]. Most of the states that contribute to the  $o(1)$  thermal probability computed by Gibbons and Hawking [29] for field theory states of static energy  $\sim 1/R$  are short wavelength states propagating near the horizon. The quantum probabilities for those states to be experimentally accessible to a detector on the geodesic, and thus within the realm in which quantum field theory has been tested, are exponentially small. States of energy  $1/R$  that *are* described by QFT are soft graviton states with wave functions spread over the entire diamond. These give an  $o(1)$  contribution to the  $o((R/L_P)^{d-2})$  entropy of dS space, and indeed the fluctuation in that entropy is  $o(1)$ . The results of [30] show that one can obtain the effective thermal spectrum for exponentially small probabilities with modular fluctuations that are of order the entropy. Thus, the universal fluctuation formula

$$(\Delta K)^2 = K, \tag{2.4}$$

is compatible with the explanation of the Gibbons-Hawking temperature in terms of constrained q-bits, *for states that are well approximated by quantum field theory*. It should be emphasized that this does not prove that the universal fluctuation formula is correct, but only that it is compatible with the explanation of semi-classical de Sitter physics by the assumption that static energy times de Sitter radius is a count of the number of constrained holographic q-bits. Short wave length near horizon states are not described by QFT in curved space-time.

### 3 AdS/CFT and Finite Causal Diamonds

Much recent work on locality in quantum theories of gravity has been focussed on the Ryu-Takayanagi formulae relating entanglement entropies in the boundary CFT to areas of surfaces in the bulk. This has been correlated with the construction of "local" bulk operators from boundary CFT operators and the tensor network/error correcting code picture of AdS space. More recent work has related these results to the formalism of algebraic quantum field theory. In the course of that, a subtle shift in emphasis has occurred, which misses an important point.

Early work [20] stressed that once one moved in to a finite global radial coordinate in AdS, one was dealing with a cutoff version of the CFT. This translates into a restriction on

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<sup>1</sup>Xi Dong, private communication.

proper times. The proper time along the geodesic at  $r = 0$  in global coordinates

$$ds^2 = -(1 + r^2/L^2)dt^2 + \frac{dr^2}{(1 + r^2/L^2)} + r^2 d\Omega_{d-2}^2. \quad (3.1)$$

is related to the time on the boundary by an infinite rescaling. For  $t < \pi L/2$ , the causal diamond for the interval  $[-t, t]$  at  $r = 0$  has finite area. After this time it touches the boundary. If we accept the view that finite  $r$  corresponds to finite cutoff, then the algebra of operators describing the causal diamond for proper time less than  $\pi L/2$ , should be finite dimensional *as long as the number of states in each node of the tensor network defining the cutoff is finite*. Susskind and the present author have argued for more than a decade that the number of q-bits in a tensor network node is one of the parameters controlling the semi-classical expansion for bulk gravity. A large number of q-bits per node is a necessary but not sufficient condition for the Einstein-Hilbert Lagrangian to be a good approximation. The results of Perlmutter [21] show that a further requirement is that the number of operators whose dimension remains finite in the large  $N$  limit must grow like  $e^{N^p}$ , where  $p$  is the power that appears in the black hole entropy formula. Furthermore, the modular fluctuations in a boundary causal diamond, must satisfy  $(\Delta K)^2 = \langle K \rangle$  to leading order in the boundary UV cutoff. This is a direct consequence of Perlmutter's calculation, for those CFTs which are well approximated by the EH action in AdS space. It agrees with the calculation of the modular fluctuation in the second paper of [35].

In [22] the authors exploited some remarkable numerical results of Evenbly and Vidal on the Tensor Network Renormalization Group (TNRG) for simple low dimensional systems. These authors formulate the construction of a tensor network for a lattice model at its critical point by writing the tensor network wave function as sequential embeddings or quantum encodings of lower dimensional Hilbert spaces into the space of the full model. The parameters of the embedding maps were determined by a numerical variational principle. Remarkably, EV found that the eigenvalues of the low dimensional "Hamiltonians", the logarithms of the embedding maps, were close to the dimensions of low dimensional operators in the CFT describing the critical behavior. In [22] we suggested that these embedding maps be thought of as the mapping of the quantum information in a causal diamond corresponding an interval  $[-t, t]$  of proper time along the geodesic at the origin, to the diamond of the interval  $[-L_p - t, t + L_p]$ . In my talk at the Princeton conference on the 20th anniversary of the AdS/CFT correspondence [23], I suggested that there would be, following the work of Evenbly and Vidal, a truncation of the operator product algebra of the CFT to low dimensional operators, corresponding to the spectrum of the embedding operators. Once  $t$  reaches  $\pi L/2$  the Hilbert space becomes infinite dimensional and a famous theorem in AQFT tells us that the operator algebra of any finite time interval is the full algebra of the CFT.

In a recent paper, Leutheusser and Liu [24] have constructed an algebra of operators corresponding to a finite proper time causal diamond in AdS/CFT, by first taking the  $G_N \rightarrow 0$  limit. In this paper, finite proper time means "finite in AdS radius units, in the limit that that radius is strictly infinite in Planck units". They argue that, in effect, this limit is being taken to all orders in the  $1/N$  or  $G_N L^{2-d}$  expansion. One of the results of their paper is that the algebra of operators corresponding to a dimensionless boundary time interval  $2w$ , has, for  $w < \pi/2$  a non-trivial commutant, which they identify with the algebra of operators

in the bulk causal diamond defined by the geodesic at the origin of global coordinates with proper time interval  $\tau \leq (\pi/2 - w)R$ .

For finite  $N$  (finite  $G_N$ ) the axioms of algebraic QFT show that the commutant of the subalgebra of any boundary time interval vanishes. The latter result should not be surprising to anyone familiar with the tensor network/error correcting code approach to the AdS/CFT correspondence. In that framework the whole point is that a finite region of a Cauchy slice in the bulk is a code subspace, whose entire operator algebra is reproduced in the larger algebra of larger shells of the tensor network. The reproduction is stable against local errors: one can omit anything less than half of the q-bits in the larger shells and still retain the information about the code subspace.

In [22] we showed how this picture was *consistent* with bulk causal structure. Crucial to that construction was the idea that any finite bulk proper time in Planck units was less than any boundary proper time in units of the CFT cutoff. The embedding maps of the tensor network renormalization group are viewed as part of the unitary transformation that maps the Hilbert space  $\mathcal{H}_{\diamond_\tau}$  into  $\mathcal{H}_{\diamond}(\tau + L_P) = \mathcal{H}_{\diamond}(\tau) \otimes \mathcal{H}_\delta$ . Here  $\mathcal{H}_\delta$  is a Hilbert space whose dimension is determined by the Bekenstein-Hawking area formula for a causal diamond of proper time  $\tau$  in *AdS* space. The relation between time on the geodesic through the origin and time in the tensor network shell at proper time  $\tau$  is

$$T_{shell} = L \tan(\pi\tau/2L_P). \tag{3.2}$$

The entropy in the shell scales like  $T_{shell}^{d-2}$ , since the radial coordinate of the shell scales like  $T_{shell}$ . Using the full symmetry group of AdS, we can choose any causal diamond of proper time  $\tau < \pi L/2$  to lie in the central node of our tensor network description of the CFT.

None of the finite dimensional operator algebras corresponding to causal diamonds in the bulk are subregion algebras or commutants of subregion algebras on the boundary. To truly understand the bulk one must impose the UV cutoff implied by a tensor network. All of this is consistent with what we have seen in the previous section. The Type III algebras of QFT are not really a good approximation to *anything* in the theory of quantum gravity on scales small compared to the AdS radius or in models with non-negative c.c. . QFT describes only low entropy constrained subspaces of the local diamond algebras of the theory of quantum gravity, whose typical states are states on the Carlip-Solodukhin (regularized) 1 + 1 CFT. They do not resemble states in a bulk QFT on small scales because, as explained in [9] most such states have large gravitational back reaction and are actually black holes. However QFT does correctly predict that the expectation value of the modular Hamiltonian  $K$  of a diamond scales like its area and that this operator has a dense set of very small eigenvalues. The conjecture of Carlip and Solodukhin makes a fairly precise statement about how this infinity is regularized.

Quantum field theorists familiar with the Wilsonian approach to renormalization may be somewhat uneasy with the last sentence of the penultimate paragraph because it is well known that regularizations of quantum field theory have non-universal features. A possible interpretation of this non-universality may have to do with the meaning of a time-like trajectory in terms of actual physical measurements. A time-like trajectory is an abstraction of the trajectory followed by a real physical detector. In a theory of quantum gravity, a real physical detector has to deal with several competing pressures. It must not be too small and



light, in order for us to be able to neglect the quantum fluctuations of its center of mass. It must not be too heavy, in order to minimize its gravitational effect on other systems whose properties it is measuring. Its information gathering and storage capacity should be as large as possible, but not so large that it collapses into a black hole. The quantum fluctuations of the center of mass are most easily dealt with. Even miniscule objects of sufficient internal complexity, like grains of dust with a volume of  $10^{-3}$  cm. have decoherent center of mass motion for times so long that they are essentially the same number expressed in Planck units as in ages of the universe. The real issue, as shown by the CKN bound, is information gathering and storage capacity. So perhaps one should say that the real reason that finite entropy, truly local physics has proven so hard to capture by rigorous mathematics, is that it is intrinsically ill defined because we cannot make a perfect detector that doesn't collapse into a black hole. We can make rigorous mathematical models of the most perfect detector one can make, but there will not be a single "correct" one, which we can verify with infinite precision. If we lived in an imaginary world with Minkowski or AdS asymptotics, and had infinite time on our hands, things might be different, but we appear to be doomed to suffer the slings and arrows of outrageous de Sitter space, and live in a mathematically imprecise world, until our local group of galaxies collapses into a black hole.

### 3.1 Phenomenological Consequences of the Fluctuation Formula

We have argued elsewhere [31] that the universal formula for fluctuations in causal diamonds forms the basis for a theory of Cosmic Microwave Background Fluctuations and Early Universe cosmology that has very few parameters and no "Transplanckian" problems. In addition, it lends support to the contention [35] [6] [37] that interferometer experiments might detect quantum gravitational fluctuations in the near future. In that context, the most important unsolved problem is to obtain a first principles calculation of the Power Spectral Density of the Fluctuations: the correlations between fluctuations in two time separated diamonds. Work on this problem is ongoing.

## 4 't Hooft Commutators and the Diamond CFT

To conclude, I want to point out a connection between the 't Hooft commutation relations, which follow from  $S_{top}$  and entropy fluctuations captured by the Carlip-Solodukhin conjecture. The 't Hooft operators  $X^\pm$  are defined on the past and future boundaries of a causal diamond, and are "canonically conjugate" to each other at equal time, where equal time means the bifurcation surface of the diamond. The Carlip-Solodukhin ansatz postulates identical CFTs living on longitudinal intervals on the past and future boundaries of the diamond. Notice that when we switch from the past to the future boundary, the role of light front time and longitudinal coordinate are exchanged.

If we imagine that the past and future boundaries are both described by the same  $1 + 1$  dimensional CFT, then natural objects that would have something like canonical commutation relations are the time and space components of a conserved  $U(1)$  current. These vanish except at coinciding points, and this fits the fact that the operators  $X^\pm$  are only at "equal

time” on the bifurcation surface of a single diamond<sup>2</sup>. The fact that they are related to the time and space components of a current matches the switch between time and longitudinal coordinate between past and future boundary.

To obtain the transverse structure of the ’t Hooft commutation relations, we conjecture that the  $U(1)$  currents are constructed as bilinears in  $1 + 1$  dimensional Dirac fields that are also spinor fields in the transverse dimensions [31]

$$X^{+/-}(\Omega) = \int f(x^\mp) \bar{\psi}_{aA}(x^\mp, \Omega) \gamma_{ab}^{0/1} (D)_{AB}^{-1}(\Omega, \Omega') \psi_{bB}(x^\mp, \Omega'). \quad (4.1)$$

$\gamma$  are the two dimensional Lorentzian Dirac matrices and  $D$  is the Riemannian Dirac operator in the transverse space.  $x^\pm$  are longitudinal coordinates on the past and future boundaries. The boundaries share one instant of time in common and the function  $f$  is chosen so that the Schwinger term in the equal time commutator between time and space components of the current integrates to 1. We then get the projection of the tensor product  $D^{-1} \otimes D - 1$  on the zero form subspace of the tensor product of two spinor bundles, which by Lichnerowicz theorem is  $\Delta - \frac{1}{4}R$ . This is not quite the ’t Hooft relation, but it is close. The mismatch suggests that the connection on the spinor bundle might not be the Riemannian connection, but contain an extra  $U(1)$  factor.

Some further subtleties: both the two dimensional CFT, and the eigenfunction expansion of the transverse Dirac operator must be cut off in order to be compatible with the principle that a finite area diamond has finite entropy. This should not bother us. The arguments of ’t Hooft and [3] are semi-classical and do not incorporate the covariant entropy bound. In addition, the two dimensional field theory cannot be a theory of free two dimensional fermions. That model is integrable and cannot incorporate the fast scrambling [38] properties of horizons. What we need are non-integrable, conformally invariant perturbations of the free fermion theory, which preserve the  $U(1)$  current algebra relations that we used above.

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<sup>2</sup>Indeed, there is a residual gauge invariance that allows us to set  $h_{--}$  to zero on the past boundary and  $h_{++}$  to zero on the future boundary of the diamond.

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