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Holographic Space-Time

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Abstract

The theory of holographic space-time (HST) generalizes both string theory and quantum field theory. It provides a geometric rationale for supersymmetry (SUSY) and a formalism in which super-Poincare invariance follows from Poincare invariance. HST unifies particles and black holes, realizing both as excitations of non-commutative geometrical variables on a holographic screen. Compact extra dimensions are interpreted as finite dimensional unitary representations of super-algebras, and have no moduli. Full field theoretic Fock spaces, and continuous moduli are both emergent phenomena of super-Poincare invariant limits in which the number of holographic degrees of freedom goes to infinity. Finite radius de Sitter (dS) spaces have no moduli, and break SUSY with a gravitino mass scaling like $\Lambda^{1/4}$. In regimes where the Covariant Entropy Bound is saturated, QFT is not a good description and HST, and inflation is such a regime. Following ideas of Jacobson, the gravitational and inflaton fields are emergent classical variables, describing the geometry of an underlying HST model, rather than "fields associated with a microscopic string theory". The phrase in quotes is meaningless in the HST formalism, except in asymptotically flat and AdS space-times, and some relatives of these.

Contents

1 Introduction

The theory of Holographic Space Time (HST) is an attempt to construct a general framework for models of quantum gravity. By quantum gravity I mean a quantum system, whose observables can, in some approximation, be computed by solving Einstein's gravitational field equations, perhaps coupled to some other classical fields.

HST shares some properties of Quantum Field Theory (QFT), and purports to be the underlying general theory, of which extant string/M-theory models are special cases. In particular, it shares with QFT the assignment of an operator algebra $A(D)$ to every causal diamond D in a Lorentzian space-time. As in QFT, the inclusion relations between operator algebras encode the causal structure of the space-time. In particular, for any pair of diamond algebras $A(D_{1,2})$, there is a common tensor factor $O(D_1, D_2)$, which consists of all of the operators localized in the largest causal diamond in the intersection of D_1 and D_2.

As in QFT, a time-like trajectory, can be characterized by a nested sequence of causal diamonds, which corresponds to a sequence of operator algebras $A(n)$, each contained in the next as a tensor factor, $A(n + 1) = A(n) \otimes P$. Geometrically these are thought of as the causal diamonds between pairs of points at larger and larger time-like separation along the trajectory. Space-time can be viewed as a Cauchy surface, labelled by a coordinate x and a congruence of non-intersecting time-like trajectories passing through x. This corresponds to a parametrized set of sequences of operator algebras $A(n, x)$.

There are two major differences between HST and QFT. The *holographic screen* (holoscreen) of a causal diamond in d dimensional space-time, is the maximal area, space-like $d - 2$ surface on the diamond's $d - 1$ dimensional null boundary. A causal diamond with small enough proper time separation between its past and future tips has a finite area holoscreen. In QFT the algebra of *any* causal diamond is infinite dimensional. In HST, a finite area diamond has a finite dimensional operator algebra, which is the algebra of matrices in a finite dimensional Hilbert space. The origin of this postulate is the Covariant Entropy Bound (CEB) of Fischler Susskind and Bousso [1].

One advantage of this postulate is that the quantum information in the sequence of Hilbert spaces gives a *definition of an emergent space-time in terms of purely quantum mechanical concepts*. That is, the collection of Hilbert spaces $\mathcal{H}(n, x)$, and overlap spaces $\mathcal{O}(n; x, y)$ defines both the causal structure of space-time, and the conformal factor. The latter is specified in terms of the areas of the holoscreens of a collection of causal diamonds, which becomes a topological cover of the space-time. The quantum theory retains the geometrical notion of time, *along each of the time-like trajectories*, and requires a specification of the topological structure of the Cauchy surface which is penetrated by the congruence of trajectories. This will be done by taking the space of labels, x to be the $0-$simplices of a $d - 1$ dimensional simplicial complex. In all extant models, the topology is taken to be that of flat $d-1$ dimensional space, which enables us to model space-time geometries with infinite spatial Cauchy surfaces with non-positive asymptotic curvature. As we will see, the formalism can also describe space-times which are the quantum version of de Sitter space and cosmologies which are de Sitter (dS) in the asymptotic future.

The second major difference between HST and QFT is in some sense more radical and has to do with the way the Hamiltonian formalism is implemented. In QFT, the Hamiltonian is traditionally written as an integral over spatial points of a Hamiltonian density. It is well known that in classical general relativity, there is no gauge covariant meaning to the energy density. Instead, the energy exists only for space-times which have time-like asymptotic Killing vectors, and is written as an integral over a surface at infinity. A standard quantum mechanics with Schrödinger evolution is obtained only when the conformal boundary of space-time contains a time-like segment. In HST, we instead assign a different Hamiltonian $H(t,x)$ to each time-like trajectory, where t is the proper time along the trajectory. Each of these quantum systems, is a complete description of the universe as it would be seen by a detector traveling along that trajectory. The key dynamical consistency condition of HST is that, when two trajectories share information, the density matrices prescribed in the overlap Hilbert space, by the time evolution in each trajectory's Hilbert space, should be unitarily equivalent to each other. This is an infinite set of conditions, at each time, and for every pair of trajectories.

The dynamical consistency conditions of HST are the mathematical formulation of the notion of *observer complementarity* [**?**], and simultaneously a radical reformulation of the concept of *many fingered time* familiar from the Wheeler-DeWitt approach to the quantization of gravity. The WD approach, in my opinion, makes no sense outside the semiclassical expansion, nor does any other approach which takes space-time geometry to be a fluctuating quantum variable. In HST quantum theory refers directly to observables that can be measured by experiments done along a single time-like trajectory. The philosophical stance of the theory is thus one of extreme positivism; one might even say solipsism. Space-time is an emergent construction, which results from an infinite set of solipsistic observers, and consistency conditions between them.

In asymptotically flat and AdS space-times, time-like trajectories which don't fall into stable black holes, have asymptotic causal diamonds whose boundary is the conformal boundary of the space-time. In such space-times the different observer Hilbert spaces are, asymptotically, just copies of each other, related by asymptotic symmetries.

The space-time metric is not a fluctuating quantum variable. Instead, the quantum variables are quantized versions of "the orientations of pixels on the holoscreen". Thinking classically for the moment, a pixel is characterized by a null direction, and a bit of transverse hyperplane orthogonal to it. This information is encoded in the Cartan-Penrose (CP) equation

$$\bar{\psi}\gamma^{\mu}\psi(\gamma_{\mu})_{\alpha}^{\beta}\psi_{\beta} = 0,$$

which implies that $\bar{\psi}\gamma^{\mu}\psi$ is null, and that ψ itself is a null plane spinor for this null vector. The CP equation is local on the holoscreen, and has local Lorentz invariance. We fix this gauge redundancy by giving a unique null direction for each point on the screen, as a consequence of which the solution of the equation is a section of the spinor bundle over the screen. For example, for a spherical screen, parametrized by a $d-1$ dimensional unit vector $\mathbf{\Omega}$, the gauge choice for the null vector is $(\pm 1, \mathbf{\Omega})$. The $\pm$ sign correspond to the same null direction, but when we discuss particles, the distinction will be that between incoming and outgoing states.

The local scale invariance of the CP equation is broken to a local Z_2 by the quantum commutation relations that we will postulate below. The Z_2 is the familiar $(-1)^F$ gauge symmetry of spin-statistics fame. The rest of the statistics gauge symmetry, the S_N which exchanges identical particles, emerges only in the particle physics limit of HST.

Quantum mechanics, in the form of the Covariant Entropy Bound (CEB) now requires both that the continuous geometry of the classical holoscreen be *pixelated* and that the (unitary) representation of the quantum algebra of single pixel variables be finite dimensional. The conventional notion of pixel may be thought of as the replacement of the algebra of functions on a manifold by a finite dimensional algebra generated by functions that $= 1$ on a single pixel and zero on all others. A more general notion of pixelation replaces the algebra of functions by a sequence of finite dimensional non-commutative algebras, which converges to the algebra of functions in an appropriate sense. The non-commutative approach, often called fuzzy geometry, can incorporate continuous isometry groups at every step.

In [2] J. Kehayias and I proposed an alternative approach to "fuzzification", which proceeds from a finite dimensional approximation to the spinor bundle, rather than the algebra of functions. This can be done in a way which preserves all symmetries, by imposing an eigenvalue cutoff on the Dirac operator, or a generalized Dirac operator, whose connection includes contributions from p-form fluxes on the manifold. Dirac fuzzification preserves all isometries of the manifold, but it also preserves the notion of covariantly constant spinor, which allows us to preserve SUSY, as well as those elements of the cohomology that are bilinears in covariantly constant spinors.

Connes [6] has argued that the general properties of the Dirac operator and its relation to the algebra of functions on a manifold, provides us with a way to generalize metrical geometry to non-commutative algebras. For physicists, this is best understood by noting that the short time expansion of the heat kernel of the square of the Dirac operator allows us to compute the metrical distance between points and a variety of curvature invariants (the Todd class). In HST, the cutoff Dirac operator on the holoscreen, tells us the number of generators in the quantum algebra of operators. This determines the area of the holoscreen, but more detailed properties of the geometry are encoded in the quantum algebra, rather than in the relation between the Dirac operator and some "fuzzy algebra of functions".

For compactifications to four dimensional asymptotically flat space-time, the appropriate commutation relations are

$$[\psi_i^A(P), {\psi^\dagger}_B^{\ j}(Q)]_+ = \delta_i^j \delta_B^A Z_{PQ}.$$

The indices i, j run from one to N, and A, B from one to $N+1$, so that the variables are elements of the two chiral spinor bundles over the two sphere, with Dirac eigenvalue cutoff $|p| \leq N + \frac{1}{2}$. P, Q label a basis of eigenfunctions of the Dirac operator on the internal manifold, with a Dirac eigenvalue cutoff K. Z_{PQ} is a corresponding basis in the bundle of differential forms. For fixed i, A the *pixel* superalgebra has a unitary representation of dimension $\mathcal{P}$. We demand that it be generated by the action of the fermionic generators on a single state. $\mathcal{P}$ is thus exponential in the number of fermionic generators.

For large K, the degeneracy of Dirac eigenvalues scales as K^D, where D is the dimension of the smooth manifold we retrieve in the large K limit. Thus the Bekenstein Hawking formula

$$\pi(RM_P)^2 \to N^2 \ln \mathcal{P} \to K^D N^2,$$

suggests an interpretation of N as proportional to the radius of the sphere in Planck units, while K is the typical linear scale of the internal manifold, in higher dimensional Planck units. The four dimensional and higher dimensional Planck scales are related by the usual Kaluza-Klein formula. Note that this worked only because the spinor bundle of a direct product manifold is a tensor product of the individual spinor bundles.

Asymptotically flat space-time with fixed Planck scale internal dimensions is obtained by taking a limit $N \to \infty$ with K fixed. This limit has several interesting features. First of all, the "internal geometry" has no moduli, since it is characterized by a finite dimensional unitary representation of a superalgebra. Moduli would arise if we also took K to infinity. In this limit there would be a number of effectively continuous ratios of integers, and in HST, this is the origin of the moduli of string/M theory.

To control the $N \to \infty$ limit, we must invoke a conformal group which acts on the two sphere. Two known ways of taking the limit lead to $SO(1,3)$ the conformal group of S^2, or $SO(2,3)$ the conformal group of $S^2 \times R$. These limits correspond to quantum gravity in asymptotically flat, or AdS space respectively. In the AdS case, the limiting theory must have a (conjugacy class of) symmetry generators, corresponding to translation along the real line in the boundary cylinder. The theory must be conformal on the cylinder, and the principle of *asymptotic darkness* [5], which says that the Bekenstein-Hawking formula determines the asymptotic density of states in AdS space, tells us that the theory behaves like a CFT. This is probably enough to construct a proof of locality of the boundary theory, which would be a derivation of the AdS/CFT correspondence from HST.

Nothing guarantees that the AdS radius of curvature is large enough so that a long wavelength quantum effective field theory in the bulk computes the important CFT correlators. It's plausible that this should only be true if the limit in which the radius of curvature goes to infinity exists. That is, the CFT should be part of a sequence or continuous family of CFTs, such that the radius can be arbitrarily large, and the limiting theory admits a description as quantum gravity in asymptotically flat space. Recent work on Mellin transformed CFT correlators makes this conjecture much more plausible [7]. Note however that we have no robust examples where the internal manifold has fixed size when the AdS radius goes to infinity.

In the would-be asymptotically flat case, $SO(1,3)$ invariance of the limiting theory of an infinite causal diamond, is NOT sufficient to guarantee Poincare invariance. If however, the internal manifold has a covariantly constant spinor[1], then there is a zero mode of the Dirac operator, whose bilinears include a constant function on the manifold. For the zero mode, we have

$$[\psi_i^A(0), {\psi^\dagger_B}^j(0)]_+ = \delta_i^j \delta_B^A.$$

[1] More generally, a spinor covariantly constant under a generalized connection which includes SUGRA fluxes, and a Dirac operator utilizing this connection.

We can define combinations of the pixel variables

$$\psi(\Omega_0) = \psi_i^A (q_A^i)_{\Omega_0},$$

such that in the limit

$$[\psi(\Omega_0), \psi^\dagger[q^*]]_+ = pq^*(\Omega_0),$$

where $q(\Omega)$ is any measurable section of the chiral spinor bundle with negative helicity. p is a positive normalization constant, which arises when taking the continuum limit to make a delta function distribution.

The conformal Killing spinor equations,

$$D_I q_{L,R} = \gamma_I^{\pm} \pi_{L,R},$$

are Lorentz covariant and their solutions transform as the left and right chiral spinor representations, q_α and $q^*_{\dot{beta}}$ of $SO(1,3)$. They are, of course, complex conjugates of each other. When we smear the delta function pixel operators with these solutions, we get a set of operators Q_α such that

$$[Q_\alpha, \bar{Q}_{\dot{\beta}}]_+ = pq_\alpha(\Omega_0) q^*_{\dot{\beta}}(\Omega_0) = p(1, \Omega_0).$$

In HST, momentum thus arises from SUSY, and asymptotically flat space is predicted to be exactly supersymmetric, in agreement with the extant evidence from string theory. This also leads to an association of SUSY breaking with the positive value of the c.c. in the real world. To understand that a little better, note that we have, so far, only demonstrated how to extract one multiplet of supersymmetric particles from the HST formalism. We get multi-particle states by taking the above described limits in $K_i \times K_i$ blocks, exploiting the oft used connection between the S_N gauge symmetry of block diagonal matrices, and that of particle statistics.

An interesting constraint arises when N is finite. We'd like to take each K_i large, in order to make our particles as localizable as possible on the two sphere. However, we'd also like to have the option of many particle states, in order to have an regime where quantum field theory is a good approximation. The optimal compromise is to take each $K_i \sim \sqrt{N}$, leading to a total particle entropy of order $N^{3/2}$. This means that most of the entropy in a finite causal diamond is not describable in terms of particles. Indeed, in a theory of gravity, we expect that the states that saturate the covariant entropy bound are mostly black holes with a horizon area that of the diamond. The maximal entropy of particle states in a finite region, which do not form a black hole, is well known to scale like $N^{3/2}$. Our matrix construction reproduces this gravitational result from simple counting. One can also follow this intuition to obtain a simple understanding of the Unruh effect in HST [8]. In our construction of the SUSY algebra, the block size K_i is proportional to the momentum. In a finite region, it's natural to take the unit of momentum to be $1/N$, and $K_i \sim \sqrt{N}$ indeed reproduces the cutoff one calculates on the momentum of typical particles in a maximal entropy state.

de Sitter (dS) space has a maximal size causal diamond, even for trajectories of infinite proper time. Following the logic of the previous paragraph, the natural cutoff on the momentum of typical particles in dS space is $M_P (R M_P)^{-\frac{1}{2}}$. SUSY is restored in the $R \to \infty$ limit, and this is the natural scale for splitting in supermultiplets. From the point of view of effective field theory, SUSY is a gauge symmetry and the breaking *must* take place by the super Higgs effect, which means that this splitting is the gravitino mass. We obtain the estimate $m_{3/2} = K\Lambda^{1/4}$ for the relationship between the gravitino mass and the cosmological constant. If we utilize Witten's idea [**?**] that large extra dimensions explain the ratio between the unification scale and the Planck scale, then we find $K \sim 10$, which leads to $m_{3/2} \sim 10^{-2}$ eV . This is (barely) consistent with experiment and leads, when phenomenological constraints are incorporated, to a rather specific model of TeV scale physics, with a small number of parameters [9].

Note also the angular momentum cutoff for gravitinos is of order $(RM_P)^{\frac{1}{2}}$, which means that they can be localized in an area $A \sim \frac{1}{m_{3/2}M_P}$ on the holographic screen. In [4] it was argued that the R symmetry violating operators, which, in effective field theory, give rise to the gravitino mass, arise from diagrams in which a single gravitino propagates to the horizon and interacts with $e^{\frac{AM_P^2}{4}}$ degenerate states there. A is the area over which the gravitino wave function is spread on the horizon. The exponential contribution to the gravitino mass is thus

$$e^{-2m_{3/2}R}e^{\frac{AM_P^2}{4}},$$

and cancels for $A \sim \frac{1}{m_{3/2}M_P}$. The holographic scaling law for the gravitino mass is the only one which leads to a consistent power law scaling. Note in particular that for smaller values of $m_{3/2}$, like the "natural" scaling $m_{3/2} \sim R^{-1}$, these R symmetry violating diagrams are exponentially large. In the HST formalism, particles with the "natural" scaling law are not localized on the holographic screen, and cannot be considered particles at all.

The scaling laws for entropy in a finite volume give us general insight into the limits of effective field theory. They tell us that only of order $N^{3/2}$ of the N^2 degrees of freedom in a finite causal diamond can be well described by quantum effect field theory (QUEFT). The rest are associated with the horizon . If the particle degrees of freedom interact strongly with the horizon degrees of freedom they are absorbed by it and cannot be considered independent particles. The QUEFT description breaks down.

In asymptotically flat space-time we can introduce a one parameter set of accelerated trajectories in any causal diamond. Recall that in HST, there will be a different Hamiltonian for each trajectory. The Hamiltonians are all time dependent, and we will concentrate on the one appropriate to the first and last instants of time in the diamond[2]. We write

$$H(N, a) = Z(a)P_0 + \frac{1}{N}V,$$

where V is an operator with a bound of order 1. P_0 is the operator for free particle propagation, which appears in the SUSY commutation relations. $Z(a) = 1$ for the geodesic, and decreases with a, becoming of order $\frac{1}{N}$ for the maximally accelerated observer. The operator $\frac{1}{N}V$ couples all of the degrees of freedom and would thermalize them in a time of order N. This leads to a thermal spectrum for the accelerated observers, with temperature increasing with the acceleration.

[2] In a time symmetric situation, the appropriate evolution operators to consider are $U(T_n, -T_n)$, propagating from the past to the future tip of a nested set of diamonds, whose size is labeled by n. We are talking about $H(N)$, defined by $U(T_N, -T_N) = e^{-iH(N)}U(T_{N-1}, -T_{N-1})e^{iH(N)}$.

In asymptotically flat space, for the geodesic observer, N increases with the proper time, and $S \equiv \lim_{N\to\infty} U(T_N, -T_N)$ consistently maps states where the particles are decoupled from the horizon to other such states [8]. S is the scattering operator. In dS space, by contrast, the proper time goes to infinity while N remains finite, the Hamiltonian asymptotes to a constant and all degrees of freedom eventually thermalize. The time averaged density matrix, averaged over a few times the dS Hubble time is maximally uncertain. The degeneracies of eigenstates of P_0 are such that this is a thermal state at the dS temperature for P_0. Once thermalization has occurred, QUEFT is only a good approximate description of rare fluctuations, in which a localized system of particles materializes spontaneously from the dS "vacuum".

However, there is another sense in which effective field theory is relevant to quantum gravity for generic states of the system. In a remarkable paper [3], Jacobson showed that Einstein's equation follows from the first law of thermodynamics and the assumption that entropy is proportional to area. Jacobson considers a general Lorentzian space-time and sets up a local Rindler coordinate system near a generic point P. Now consider an accelerated trajectory, with acceleration that we will eventually take to infinity. Define the energy to be the energy as viewed by an observer following that trajectory, obtained by doing the integral of the T_{00} component of the stress tensor in the accelerated coordinates. Use Unruh's formula for the temperature as a function of the acceleration, and Raychauduri's equation to describe the change in area transverse to the accelerated trajectory. Jacobson shows that $dE = TdS$, becomes $k^{\mu\nu}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - 8\pi G T_{\mu\nu}) = 0$, where k^{μ} is the limiting null velocity of the accelerated observer. Since this is true at every point and for every null k^{μ}, we get Einstein's equation, *apart from a possible cosmological term*, which vanishes when contracted with the null vector.

Jacobson's argument shows that the classical Einstein equations are the local hydrodynamics of any quantum system obeying the Bekenstein-Hawking relation between entropy and area. A full set of hydrodynamic equations require further specification of the stress tensor, either by postulating an equation of state or introducing other classical fields. Jacobson argues that these fields, and in particular the space-time metric, should not necessarily be thought of as quantized fields. We do not quantize the hydrodynamic equations of generic quantum systems.

This argument meshes perfectly with the formalism of HST. In HST the entropy/area connection *defines* space-time in terms of quantum mechanics. In all extant examples satisfying the HST consistency requirements, the large causal diamond limit of the dynamics can indeed be put in one to one correspondence with a solution of Einstein's equations. In some examples the stress tensor satisfies an equation of state, while in others, there is an additional classical scalar field [10]. In particular, the inflaton field in the holographic model of inflation, is such a classical hydrodynamical field. Its fluctuations are introduced to match a particular ansatz for the Hamiltonian of an underlying HST model, and their origin is thermal, rather than quantum mechanical. As a consequence of decoherence, it is of course impossible to verify observationally the putative quantum origin of CMB fluctuations.

Jacobson's argument also fits with the long series of observations I have made over more than a decade, which show that the idea borrowed from QFT, that solutions of a classical field theory with different asymptotics, and different values of the c.c., correspond to different states of the same Hamiltonian quantum mechanics, has no place in a quantum theory of gravity. Those arguments were based on Matrix Theory, AdS/CFT, the analysis of Coleman-

DeLucia tunneling, an analysis of production of regions of "metastable vacuum", and an analysis of the quantum meaning of the Wheeler DeWitt equation. Jacobson's argument shows us that gravitational field equations follow from local thermodynamics, but that the value of the c.c. must be appended as an infrared boundary condition.

Jacobson's thermodynamic effective field theory (THEFT) is a very general emergent property of systems of quantum gravity, while the more conventional effective quantum field theory (QUEFT) is restricted to asymptotically flat, and large radius dS and AdS universes, and others which have morally similar asymptotics. In the dS case QUEFT is only a good description over time scales less than or of order the Hubble time. On longer time scales, localized excitations thermalize with the horizon degrees of freedom, which are not well described (except thermodynamically) by QUEFT.

The principles of HST have profound consequences, which should eventually lead to experimental verification or falsification of the theory. In particular, the relation between the c.c. and the scale of SUSY breaking, gives a very low value for the masses of charginos. These particles *must* be found at the LHC, with masses not too far above the current Fermilab bound. It is likely that TeV physics is described by some version of the Pyramid Scheme [9], though it is not clear how much of the structure of that model will be revealed at the LHC. Dark matter is apt to be a hidden sector particle charged under a new kind of baryon number. It's likely to be fairly heavy, not a thermal relic, but produced by an asymmetry, and to have either a magnetic or chromo-magnetic moment.

HST also gives us a completely non-singular, quantum description of the very early universe, which can closely match the results of inflationary models, and, unlike those models, explains the low (localized) initial entropy of the part of the universe we see. It accommodates an anthropic explanation of the value of the c.c. and the magnitude of initial density fluctuations, while allowing us to argue that all other low energy parameters are determined in terms of the c.c. . That is, the HST model of the world we observe may be fairly uniquely fixed by a few cosmological parameters. It is not yet clear whether the HST model of the very early universe has observational signals sufficiently different from classic inflation models, to allow the two to be distinguished.

A huge amount of work remains to be done on all aspects of the HST formalism and phenomenology. I hope this essay will convince some young talented people to think about it.

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References

[1] W. Fischler and L. Susskind, "Holography and cosmology," `arXiv:hep-th/9806039`. R. Bousso, "A Covariant Entropy Conjecture," *JHEP* **07** (1999) 004, `arXiv:hep-th/9905177`. R. Bousso, "Holography in general space-times," *JHEP* **06** (1999) 028, `arXiv:hep-th/9906022`.

[2] T. Banks and J. Kehayias, “Fuzzy Geometry via the Spinor Bundle, with Applications to Holographic Space-time and Matrix Theory,” Phys. Rev. D **84**, 086008 (2011) [arXiv:1106.1179 [hep-th]].

[3] T. Jacobson, “Thermodynamics of space-time: The Einstein equation of state,” *Phys. Rev. Lett.* **75** (1995) 1260–1263, `arXiv:gr-qc/9504004`.

[4] T. Banks, “Breaking SUSY on the horizon,” hep-th/0206117.

[5] T. Banks, Talk at the 2001 Festschrift for D. Gross, KITP, UCSB.

[6] A. Connes, *Noncommutative Geometry.* Academic Press, 1994.

[7] A. L. Fitzpatrick, J. Kaplan, J. Penedones, S. Raju and B. C. van Rees, “A Natural Language for AdS/CFT Correlators,” JHEP **1111**, 095 (2011) [arXiv:1107.1499 [hep-th]].

[8] T. Banks, W. Fischler, *The Unruh Effect in Holographic Space-time*, manuscript in preparation.

[9] T. Banks and J. -F. Fortin, “A Pyramid Scheme for Particle Physics,” JHEP **0907**, 046 (2009) [arXiv:0901.3578 [hep-ph]].

[10] T. Banks and W. Fischler, “Holographic Theories of Inflation and Fluctuations,” arXiv:1111.4948 [hep-th].